PROBLEM BASED LEARNING IN BASIC PHYSICS - XI

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[In this article, eleventh in the series of articles, we present problems for a problem-based learning course from the area of geometrical optics. We present the learning objectives in this area of basic physics and what each problem tries to achieve with its solution.]

Abstract

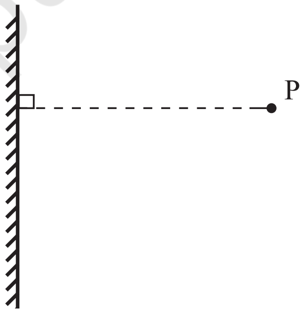
In this article, eleventh in the series of Problem-based Learning in Basic Physics, we present problems on geometrical optics. Methodology and philosophy of selecting these problems are already discussed. In this article we present problems from which students apart from solving actual problems can carry out an activity of geometrically constructing ray diagrams to locate where the image is formed. This activity, as we found, gives students good insight into not only the process of image formation but also how the rules for image tracing are arrived at.

Learning Objectives

1. To understand how the rules for image tracing are arrived at.
2. To be able to trace the rays to see where the image is formed.
3. To be able to use sign conventions in problems of geometrical optics for locating image and its nature.

Problems

1. **Ray Tracing for Finding Image due to a Point Object in Front of a Plane Mirror**. Consider a point object in front of a mirror. We need to find the position of image formed by the mirror. We will use the laws of reflection in constructing ray diagram.



1 A. Additional Activity: Once students have succeeded in the preceding activity, they should be able to draw the diagram for the following problem: If two mirrors have an angle θ (in degree) between them, they form number (n) of images given by $n = \frac{180}{\theta} - 1$. Thus, if we take two mirrors at an angle of 90° between them, they should form three images. Make students construct a ray diagram to discover formation of two images. Even 120^0 makes a good activity.

2. Consider the Problem of Finding Image Due to a Curved Mirror

(i) A convex mirror and

(ii) A concave mirror. In both the cases we consider point objects off the principal axis.

Further Activity for Convex Mirror

Take an extended object PQ in front of the convex mirror. Construct a ray diagram to locate images of points P and Q. Now guess how the image of an extended object PQ would appear.

Calculation

2 A: A 4.5 cm needle is placed 12 cm away from a convex mirror of focal length 15 cm. Give the location of the image and the magnification. Describe what happens as the needle is moved farther from the mirror[4].

2 B: A small candle, 2.5 cm in size is placed at a distance of 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to obtain a sharp image? Describe the nature and size of the image. If the candle is moved closer to the mirror, how should the screen be moved?

Further Activities for Concave Mirror

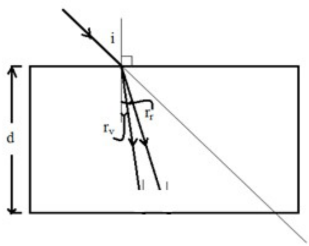
(i) For a concave mirror, we take $f = R/2$, i.e. the focal point is midway between mirror and centre of curvature. You may take two rays parallel to the principal axis. At the point of incidence, draw a normal to the mirror (radius vector). Construct reflected rays and see where they meet or appear to meet. This point will be in the focal plane. If this point is on the principal axis then it is the focal point. Can you establish $f = R/2$ using your ray diagram or otherwise? Do you find any difference between your observation and the accepted result?

(ii) You may construct the image of an extended object (say a pin) placed perpendicular to the principal axis at different positions. Say,

(a) $u = \infty$ (b) $u > 2f$ (c) $u = 2f$ (d) $f < u < 2f$
(e) $u = f$ (f) $u < f$

3. Refraction at a Plane Surface

Take a slab of thickness d = 4 cm and of material having reflective index 1.7 for violet and 1.5 for red. (These numbers are hypothetical and taken for drawing purpose. In practice, μ for violet is only slightly greater than that for red.) Draw a ray of white light (consisting of all colours from violet to red) incident at an angle of incidence 53° on the slab as shown. Calculate the angle of reflection for violet and red and draw these rays travelling in the glass slab. They all emerge at an angle 53° in air but separated. Measure separation between them. Check your answer with that obtained from $\Delta S = (\mu_V - \mu_R) d \tan(r_{mean})$.



3 A: Further Activity After you have successfully completed the above mentioned activity, repeat a similar ray diagram construction for an equilateral prism of side 4 cm. Find δ_{Violet} and δ_{Red} from your diagram. You may take $\mu_{Yellow} = 1.6$ and find δ_{Yellow}. Calculate. $\frac{\mu_V - \mu_R}{\mu_Y - 1}$. How does this value compare with your $\frac{\delta_V - \delta_R}{\delta_Y}$ based on measured values of δ? Discuss if you find any discrepancy.

4. Refraction at Spherical Surface

To understand image formation due to a spherical surface separating two media, consider a glass sphere (μ = 1.5) of radius 5 cm and having a tiny air bubble at a distance 2 cm from the centre.

We want to construct a ray diagram to know the position of the image when viewed along the diameter passing through the bubble. As seen from two different ends of the diameter, rays which reach the surface as they emerge out in the air, bend away from the normal (radial direction, except the rays travelling along the diameter, which will be travelling undeviated) and will appear to have started from I_1 and 1_2 respectively.

Use formula $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ ($\mu 1$ = 1.5 and $\mu 2$ = 1 in this case). We need to calculate positions of the images I_1 and I_2. Check if your calculated positions match with schematic diagram positions.

[**Sign Convention**: For first surface (on left), ray from the bubble travels from right to left which becomes positive direction. Distances measured υ and R are to the right of the surface and hence are to be substituted with negative sign and value of υ is to be interpreted accordingly for position of the image. Second surface is left as homework.]

Further Activity

Imagine a spherical fish tank. A fish is looking at an insect which is outside the tank. Take suitable parameters and construct your ray diagram to locate insect position as seen by the fish. Check your obtained position by calculating image position from your equation.

5. Image Formation Due to a Lens

For a thin lens, we consider these points of importance. Two focal points are such that if light is incident parallel to principal axis, then after refraction it will pass through the focal point on the other side or will appear to come from the focal point.

Rules

- Optic centre is the centre of the lens through which light ray passes undeviated.
- If a ray of light passes through the focal point or is directed towards it, then it emerges parallel to the principal axis of the lens.

All these happen in an approximation in which we assume rays to be paraxial (i.e. they make small angles with the principal axis) and lens is thin (i.e., thickness of the lens is negligible in comparision with distance we deal with for studying formation of image).

5 A: Carry out a similar ray tracing for concave lens for different positions of objects.

5 B: Repeat similar diagram for a convex lens for object at distances, say

(a) $u = \infty$ (b) $u > 2f$ (c) $u = 2f$ (d) $f < u < 2f$

(b) $u = f$ (f) $u < f$

6. Prism

Consider an equilateral prism made of material having refractive index 1.63. Let a monochromatic ray of light be incident on the first surface at an angle of incidence 37°.

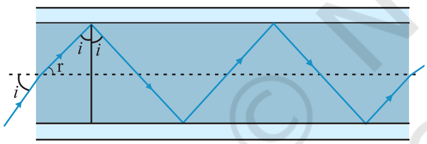
Trace the path of the ray and find angle of deviation.

7. Combination of Lens and Mirror

A convex lens of focal length 10 cm is placed in front of a convex mirror. Both are coaxial and the lens is 10 cm from the apex of the mirror. When an object is placed on the axis at a distance of 12 cm from the lens, it is found that the image coincides with the object. Calculate the focal length of the convex mirror.

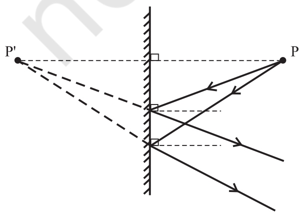
8. Total Internal Reflection[4]

The figure below shows a cross-section of a 'light pipe' made of a glass fiber of refractive index 1.68. The outer covering of the pipe is made of a material of refractive index 1.44. What is the range of angles of the incident rays with the axis of the pipe for which total internal reflection inside the pipe takes place, as shown in the figure. What is the answer if there is no outer covering of the pipe?



Solutions

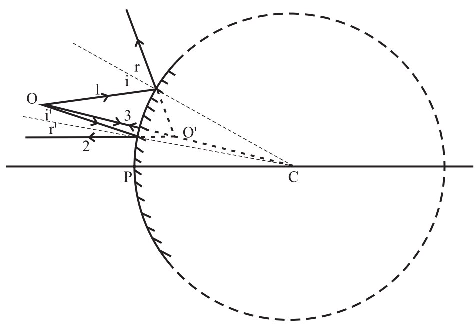
1. Take any two rays originating from P directed towards the mirror. Due to the mirror the rays are reflected as per law of reflection, that is $i = r$. For an observer, as shown in the figure, they appear to come from P'. P as seen in mirrors appears to be at P. This is the image of P.



You may ask students to repeat this exercise with any number of more rays. [*Hint*: Take one ray along normal from P to mirror. Take two rays on different sides of the normal]. Convince yourself (if your drawing is accurate and each ray obeys $i = r$) that all the rays appear to originate from P' due to reflection.

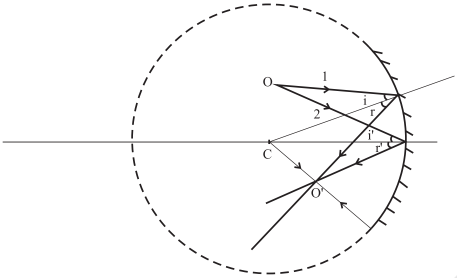
2. Reflection from a Spherical Surface

Convex Surface: Consider a convex surface. (You may take a circle of appropriate radius. Part of the circle can be treated as a convex mirror as shown.) This surface can be thought of a sphere of same radius with centre at C. Since C is centre, any line joining C to any point on the mirror will be normal to the spherical surface. Take a ray (no.1) starting from point object O to be incident on the convex mirror and draw a reflected ray as per law $i = r$. Take another ray directed to the pole and reflected similarly. Both these rays appear to come from O'. You may take more rays and convince yourself that after reflection they will appear to come from O'. For example, take a ray directed towards C, the centre of curvature. This ray is incident normally and will retrace its path backward after reflection and will still appear to be coming from O'. Thus O' is the image of point object O due to convex mirror.



Concave Mirror

Repeat reflection at the surface (Hypothetical drawn with dotted line) again supposed to be passing through activity now for a concave mirror as described for a convex mirror. A direct ray passing through C after same point O′. Any number of rays directed towards mirror thus pass through O′ which is now the image of O due to the mirror.

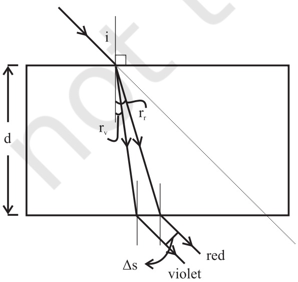


You may change the position of O for each of the mirrors placed — plane, convex and concave and redraw these ray diagrams to understand image formation due to mirrors.

2 A. Calculation: v = 6.7cm. Magnification = 5/9, i.e., the size of the image is 2.5 cm. As u → ∞; v → f (but never beyond).

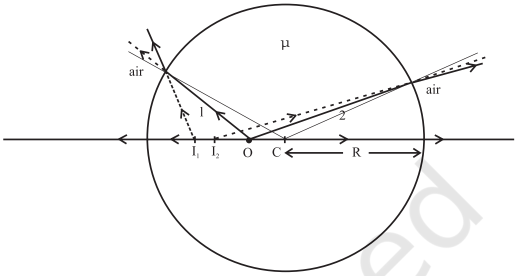
2 B. v = –54 cm. The image is real, inverted and magnified. The size of the image is 5.0 cm. As u → f, v → ∞; for u < f, image is virtual.

3.



Can you explain why in real life when we see objects through glass windows, we do not see separated colours?

4.

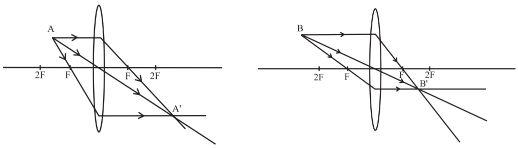


5. Consider two different off axis points A and B as shown in below. To find the images of these points due to a lens we draw three rays in the following manners.

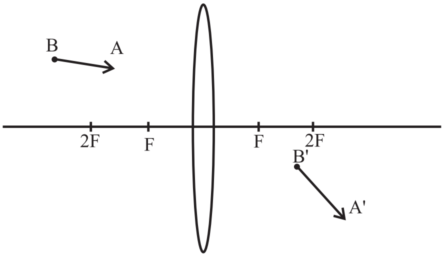
(i) Draw a ray parallel to principal axis starting from A which after refraction passes through focal point F on the other side.

(ii) A ray passing through the optic centre, that passes undeviated.

(iii) A ray passing through the focal point before passing through the lens emerges parallel to the principal axis.



(iv) The point where these three rays converge will be the image position A′ and/or B′. If we have an extended object AB, the corresponding image formed will be A′ B′.



Note: We can have infinite rays starting from A and converging to A'. The rules for ray tracing for all the rays except the three mentioned above are not so obvious. However three rays are sufficient to trace the position of the image. If by some reasons part of the lens is covered, we still have full image (only lesser in intensity). As in reality infinite number of rays starting from A and pass through the lens to form image at A'. Blocking of the lens only means blocking some of these infinite rays.

6. For equilateral prism $A = 60^\circ$

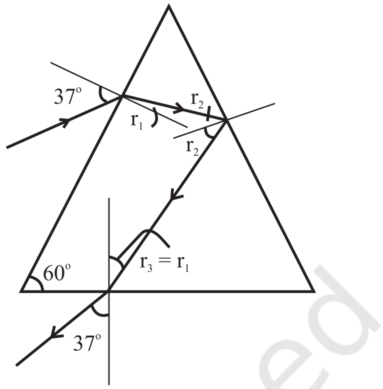
Angle of refraction at the first surface $r_1 = \sin^{-1}(\sin i/\mu) = 21.6670^\circ$

This makes angle of incidence on the second surface $r_2 = A - r_1 = 38.3329^\circ$

Thus angle of emergence from the second surface $e = \sin^{-1}(\mu \sin r_2) = \sin^{-1}(1.0109) \rightarrow$ is indeterminate.

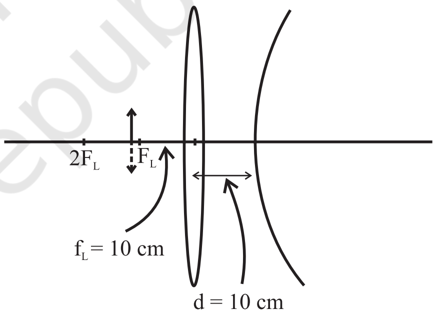
Critical angle $C = \sin^{-1}(1/\mu) = 37.8427^\circ$

Clearly $r_2 = 38.3329^\circ > C$ which means total internal reflection occurs at the second surface. In this case there will be no emergent ray and the internally reflected ray will be incident on the third surface at an angle of $r_3 = A - r_2 = 21.6670^\circ$ to emerge from the 3rd surface at an angle of emergence $e' = 37^\circ$. Clearly at the second surface $\delta_2 = 180 - 2\,(38.3329) = 108.333^\circ$ and $\delta_3 = \delta_1 = 15.333^\circ$



Thus the deviation suffered by the ray will be (referring to the figure) $\delta = 15.333 + 108.333 + 15.333 = 139^\circ$.

7.



For lens f = 10 cm and u = 12 cm. With appropriate sign convention, we get v = 60 cm to the right of the lens. This image acts as an object for the mirror and we take object distance u_m = 50 cm (since distance between lens and mirror is 10 cm).

The image formed by the mirror acts as an object for the lens which forms its final image.

Since final image for the lens is to its left at 12 cm, appropriate sign convention gives object distance as 60 cm to the right. Which means it was at 50 cm to the right of the mirror.

Thus, for mirror, object and image are at the same location. This means for mirror, 2f = 50 cm and its focal length is 25 cm.

8: $\sin i_c = \frac{1.44}{1.68}$ which gives $i_c' = 59^o$. Total internal reflection takes place when $i > 59^\circ$ or when $r < r_{max} = 31^\circ$.

Now $\frac{\sin i_{max}}{\sin r_{max}} = 1.68$ which gives $i_{max} = 60^\circ$.

Thus all incident rays of angles in the range $0 < i < 60^o$ will suffer total internal reflection in the pipe. (If the length of the pipe is finite, which it is in practice, there will be a lower limit on i, determined by the ratio of the diameter to the length of the pipe.)

If there is no outer coating ,

$i_c = \sin^{-1}\left(\frac{1}{1.68}\right) = 36.5^o$. Now, $i = 90^\circ$ will satisfy $r = 36.5^\circ$ and $i' = 53.5^\circ$ which is greater than i_c. Thus all incident rays (in the range $53.5^\circ < i < 90^\circ$) will suffer total internal reflection.

References

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