A Comparative Study on Effectiveness of Problem-Solving Method, Vedic Mathematics and CRA Approach on Solvinng Mathematical Problems at the Primary Level

Ghanashyam Sahu, Garudadhwaja Barik, and Aloklaxmi Behera*

Abstract

This research aimed to determine the effectiveness of three methods—Problem-Solving, Vedic Mathematics, and the Concrete-Representational-Abstract (CRA) Approach—in improving the mathematical problem-solving abilities of primary-level students in India, aligned with the National Education Policy, 2020. Students in Grade III were assigned to one of three groups (Group A: Problem-Solving, Group B: Vedic Mathematics, Group C: CRA Approach) and received a week-long teaching intervention. The study included 60 students who were similar in their pre-test performance but diverse in terms of gender, social category, and parental background. Results showed that all three teaching approaches had a significant impact on post-test scores, but the CRA Approach demonstrated superior effectiveness (Mean=16.40), compared to Problem-Solving (Mean=12.47) and Vedic Mathematics (Mean = 11.80). Based on stakeholders' feedback, the CRA Approach displayed exceptional strengths in promoting joyful learning (2.95), fostering deep understanding (2.90), and facilitating real-life application (2.25). These findings make a strong case for the integration of the CRA Approach into elementary mathematics education and for further refinement of current teaching methods. Additionally, future research should explore the long-term effects of the CRA Approach, expand sample sizes, investigate teacher training methods, and explore interdisciplinary connections. Notably, this study emphasizes the significant impact of teaching practices on mathematics proficiency and offers valuable insights for educators, policymakers, and curriculum developers. Ultimately, it presents valuable avenues for future research and advancement in the field of mathematics education.

Keywords*: Mathematics Education, Problem Solving, Vedic Mathematics, CRA Approach, Teachers Training*

JEL Classification Code*:* I2, C4, C9

Introduction

The NEP 2020 envisions transforming the Indian education system by emphasizing holistic development, critical thinking, and application-oriented learning (NEP, 2020). In this context, selecting appropriate teaching methods that align with the policy's objectives becomes crucial (Napper, 2012). Mathematics education at the primary level serves as the cornerstone for the cognitive development, problem-solving skills, and analytical thinking abilities of children (Harel and Sowder, 2005; Son, Darhim, and Fatimah, 2020). The process of learning mathematics involves both traditional (Huda, Florentinus, & Nugroho, 2020) and innovative pedagogy (Freudenthal, 2002; Yilmaz, 2020) aimed at enhancing understanding and mastery of mathematical concepts. In recent times, the Problem-Solving Method (Polya, 1985; Tambychik & Meerah, 2010; Badger et al., 2012) Vedic Mathematics (Day & Tan, 2022), and the Concrete-Representational-Abstract

(CRA) approach (Mercer & Miller, 1992; Misquitta, 2011; Milton et al., 2019; Reyes, 2021) have gained significant attention by the practitioners due to their potential to facilitate effective learning experiences in mathematics education.

The Problem-Solving Method is an instructional strategy that encourages students to engage with mathematical problems actively, fostering critical thinking, logical reasoning and creativity (Lester & Kehle, 2003; Zakaria, Matt & Khalid, 2019; Aljalahma, 2023). It emphasizes understanding the problem, devising strategies, and applying mathematical concepts to derive solutions, and a deeper understanding of mathematical concepts rather than rote memorization (Chin & Chia, 2004; Prathana, Suwimon & Siridej, 2014; Robert, Nganga & James, 2022). Vedic Mathematics, based on ancient Indian mathematical principles, offers alternative techniques and shortcuts for arithmetic and algebraic calculations (Shukla et al, 2017; Sharma, 2014; Mala, 2023). This method is known for its simplicity, speed, and efficiency in solving mathematical problems, employing mental calculations and innovative techniques to expedite computation processes (Muehlman, 1998; Tiwari et al, 2008; Dhivyadeepa & Govindarajan, 2013).

The Concrete-Representational-Abstract (CRA) approach, grounded in the principles of constructivist theory, scaffolds learning by progressing from hands-on experiences with concrete materials to visual representations and then to abstract mathematical concepts (Matthew, 2006; Corey & Kimberly, 2018; Nugroho & Jailani, 2019). It aims to build a solid foundation by allowing students to manipulate physical objects, transition to pictorial representations and finally understand abstract mathematical concepts through structured stages. While existing studies have highlighted the effectiveness of these methods in isolation, there is a lack of research comparing their relative effectiveness. This comparative study seeks to explore and evaluate the effectiveness of these three methodologies – the Problem-Solving Method, Vedic Mathematics, and the CRA approach – in enhancing the mathematical problem-solving skills of children at the primary level. This study aims to shed light on which approach or combination thereof yields the most significant and sustainable improvements in problem-solving abilities among primary-level students. The study involves implementing these methods within controlled classroom settings, observing students' performances, analysing their problem-solving strategies and evaluating the overall impact on learning outcomes. Further, it also aims to identify the strengths, weaknesses, and potential areas of improvement for each approach and provide valuable insights into designing more effective mathematics teaching strategies for teachers and policymakers. The effectiveness of these methodologies in primary-level mathematics education could shape pedagogical practices and foster a solid mathematical foundation essential for academic success and real-life problem-solving skills of children studying in early-grades.

Review of Literatures

In the realm of mathematics education, the choice of an appropriate teaching method holds paramount significance in shaping students' understanding, problem-solving skills, and overall performance (Shellard & Moyer, 2002). With the recent implementation of the National Education Policy (NEP) in 2020, there is a growing emphasis on innovative and effective pedagogical approaches that foster conceptual clarity, critical thinking, and application-oriented learning at the elementary level. Furthermore, Pratham's Annual Status of Education Report, 2022 and National Achievement Survey, 2021 of the Government of India consistently highlight the need to improve students' mathematical learning outcomes. By understanding the impact of different teaching approaches, this study aspires to contribute to the ongoing efforts to enhance the quality of mathematics education. As India strives to enhance its education system, it is essential to explore and evaluate various teaching methodologies to ensure students' holistic development and mastery of mathematical concepts (Feb 9, 2021, India Today). This comparative study aims to investigate the effectiveness of three distinct teaching approaches in mathematics, namely the Problem-Solving Method, Vedic Mathematics, and the Concrete-Representational-Abstract (CRA) Approach, on solving mathematical problems at the elementary level. The study's significance is underscored by the changing educational landscape in India and the increasing focus on developing students' mathematical abilities in alignment with the NEP 2020 objectives.

The Problem-Solving Method equips students with versatile strategies and techniques to approach different types of mathematical problems effectively (Liljedahl, Trigo, Malaspina, & Bruder, 2016). It emphasizes logical reasoning and creative thinking, fostering the development of problem-solving skills that are crucial for success in academic and real-life scenarios (Kirschner et al., 2011; Lochhead & Zietsman, 2001; Hair, Anderson, Tatham & Black, 1998). Similarly, Vedic Mathematics, rooted in ancient Indian mathematical systems, offers sutras and sub-sutras to expedite problem-solving and calculation (Mala, 2023). It is known for its efficiency in quick computation and mental mathematics, making it a popular approach among some educators and parents (Raikhola, Panthi, Acharya & Jha, 2020). On the other hand, the CRA Approach introduces abstract mathematical concepts through concrete manipulatives, gradually transitioning to more abstract representations (Hinton & Flores, 2022). This approach nurtures deep understanding and connections between mathematical ideas, facilitating application and retention of knowledge (Witzel, 2005). Mathematical problem-solving is a fundamental pillar of mathematics instruction, playing a crucial role in preparing students to thrive in modern society. It serves as a platform for students to apply their understanding of mathematical concepts, bridging the gaps between isolated pieces of knowledge and fostering a deeper conceptual comprehension of mathematics as a discipline (Lester and Cai, 2016). Some researchers propose that mathematics itself is essentially a science of problem-solving, dedicated to the development of theories and methodologies for effectively addressing various problem types (Hamilton, 2007; Davydov, 2008). By nurturing problem-solving skills in students, educators empower them to approach real-life challenges with a structured and analytical mindset, providing them with invaluable tools for success in their personal and professional lives.

Objectives of the Study

(a) To assess the effectiveness of three distinct teaching approaches in mathematics: Problem Solving, Vedic Mathematics and the CRA (Concrete-Representational-Abstract) approach.

(b) To compare the post-test scores of participants after undergoing different teaching methods.

(c) To determine the impact of participants' pre-test scores on their post-test performance across the various teaching methods.

(d) To identify the strengths and weaknesses of each teaching approach in terms of deep understanding, real-life application, creativity development, quick problem-solving, and material retention.

Hypotheses of the Study

(a) The intervention plan, which includes three different teaching approaches in mathematics (Problem Solving, Vedic Mathematics, and the CRA approach), has a significant impact on participants' post-test scores after intervention.

(b) There are no significant differences in the post-test scores among the three teaching approaches (Problem Solving, Vedic Mathematics, and CRA approach).

(c) Participants' pre-test scores have no significant influence on their post-test performance across the different teaching methods.

(d) The CRA approach is perceived as the most effective approach in enhancing participants' mathematical abilities compared to Vedic Mathematics and Problem Solving.

Research Methodology

The study adopted a pre and post-test experimental design. The experimental design was planned to assess the study of mathematical abilities of three different approaches in teaching mathematics to Class III students. Participants were divided into three groups (Group A, Group B, and Group C) based on homogeneity in pre-test results and heterogeneity in gender, social category, and socio-economic parental background. The Levene Statistic (Homogeneity of Variance Test) was used to assess whether the variances of pre-test scores for the three different teaching approaches were approximately equal. Each group underwent a distinct teaching approach (Vedic Mathematics, Problem Solving, and CRA) for a week. Post-test scores were measured to evaluate the impact of the teaching approaches on participants' mathematical abilities using analysis of variance (ANOVA). Further ratings were obtained from parents regarding the effectiveness of each teaching approach in various areas such as time taken to solve problems, real-life application, creativity development, and material retention.

Table 1. Intervention Plan Participants

Pre-Test	Group	Intervention	Post Test
A1	Group A	X1= Problem Solving	A2
B1	Group B	X2= Vedic Mathematics	B2
C1	Group C	X3=CRA approach	C2

The study was conducted in three government-run schools located in a semi-urban area of Kalahandi District where the students' performance in arithmetic was found to be relatively low as per National Achievement Survey, 2021. The schools cater to students from diverse socioeconomic backgrounds. The intervention was conducted in six classrooms, with 60 students aged 8-9 years, across different schools adhering to the state curriculum. Teachers in these schools were trained with the support of DIET and the researchers. The researchers played a crucial role in deploying these teaching techniques and building rapport with teachers and parents for track the progress for feedback, ensuring a smooth implementation of the study.

The intervention plan involves three different groups (Group A, Group B and Group C), each focusing on a distinct teaching approach. The pre-test and post-test were structured to evaluate students' understanding of basic mathematical concepts at varying levels of difficulty. The pre-test, which included 20 questions, focused on fundamental operations and simple word problems. For example, students were asked, "If you had 15 apples and bought 13 more, how many apples did you have in total?" In contrast, the post-test also consisted of 20 questions but with increased complexity to assess a deeper understanding of the concepts. For instance, one post-test question was, "If you had 10 toffees and received 6 each from Rabi, Kishor, and Gobind, how many toffees did you have in total?" These questions required students to apply their knowledge more comprehensively, thus evaluating their ability to handle more complex and integrated mathematical tasks in post-intervention test.

The groups were formed based on the homogeneity in pre-test results but heterogeneity in gender, social category and relatively same socio-economic parental background using matching group method.

Table 2. Similarity among Groups

	Mean Score before Intervention of various approaches in Teaching Mathematics						
			Levene Statistic for Similarity among Groups				
Approaches to Teaching Mathematics	Mean	Std. Deviation	Pre-Test before Intervention	Levene Statistic	df1	df2	Sig.
Group A	8.40	2.644	Based on Mean	.648	2	57	.527
Group B	8.00	2.938					
Group C	7.35	3.117					

The above table revealed that the mean score of the groups varies from 7.35 to 8.40. Further, the study used Levene Statistic before intervention for three different teaching approaches in mathematics to assess whether the variances of different groups' data were similar or not. Based on the Levene Statistic (F=0.648 with df=2, df=57) and its associated p-value (0.527>0.05), it is evident the variances of pre-test scores for the three different teaching approaches (Groups A, B, and C) are not statistically different. It reveals that the groups are relatively similar to their mathematical ability before the intervention.

About the Intervention

Intervention 1 (X1): This approach deployed various teaching strategies and techniques to approach and solve different types of problems effectively. This approach focuses on developing students' ability to solve mathematical problems through the application of concepts to real-life situations. It emphasized understanding the problem, devising a plan, carrying out the plan, and evaluating the solution. For example, students are tasked with calculating the total cost of fruits within a budget of Rs. 20. The fruit prices are: Apple - Rs. 10, Banana - Rs. 2, and Orange - Rs. 5.

Implementation

- **Understand the Problem**: Students identified the prices of each fruit and recognize the budget constraint.
- **Devise a Plan**: Students explored various combinations of the fruits that would sum up to Rs. 20 or less. They considered which fruits to buy and in what quantities.
- **Carry Out the Plan**: Students calculated the total cost for each fruit combination. For example, if they choose 1 Apple (Rs. 10) and 2 Bananas (Rs. 4), the total cost is Rs. 14. They repeated this for other combinations.
- **Evaluate the Solution**: Students checked if their selected combination(s) fit within the Rs. 20 budget and verify that it satisfied the budget requirement.

This approach allows students to practice mathematical operations such as addition and subtraction while also engaging in practical budgeting and decision-making.

Intervention 2 (X2): This approach uses sutras (aphorisms) and sub-sutras for various mathematical operations of ancient Indian system of mathematic to solve the problem that simplifies calculations with specific techniques for faster mental arithmetic. For instance, to multiply 97 by 96:

Implementation

Step 1: Apply the Vedic Mathematics formula for numbers close to 100: (100−97) × (100−96).

Step 2: Subtract each number from 100, resulting in 3 and 4.

Step 3: Multiply these results: 3×4=12.

Step 4: Subtract one of the original numbers (97 or 96) from 100, then add the difference to the other number: 97−4=93 or 97 - 4 = 93.

Step 5: Combine these results to get the final answer: (93×100) +12= 9300 + 12=9312

This approach aids students in performing quick mental calculations and developing a strong numerical sense.

Intervention 3 (X3): This instructional strategy introduced abstract mathematical concepts through concrete manipulative, gradually transitioning to more abstract representations. The CRA Approach is a three-step instructional strategy designed to deepen students' understanding of mathematical concepts by progressing from concrete manipulatives to representational diagrams and finally to abstract symbols.

Scenario: Students are learning about division.

Implementation

- **Step 1 (Concrete Stage)**: Use physical objects, such as counters or base-ten blocks, to demonstrate division. For example, to illustrate 12 divided by 3, distribute 12 counters into 3 equal groups, showing that each group contains 4 counters.
- **Step 2 (Representational Stage)**: Draw pictures or diagrams to represent the division problem. For instance, draw 12 dots and group them into 3 equal sets, illustrating that each group contains 4 dots.
- **Step 3 (Abstract Stage)**: Use numerical symbols and equations to represent the division problem. For example, write 12÷3=4 to represent that dividing 12 by 3 yields 4.

Duration of the Intervention: Each group received a one-week intervention using one of the three teaching methods. The one-week intervention period was selected to understand the pre-instruction testing effects which generally range from few days to few weeks aiming to provide a preliminary assessment of the impact of different teaching approaches. While a longer intervention might yield more comprehensive results, the one-week period allows for an initial evaluation of the methods' effectiveness. This duration observed noticeable changes in post-test scores in previous studies (Kliegl et al., 2022; Janelli and Lipnevich, 2021; Beckman, 2008).

Results and Discussion

The Results and Discussion section presents a detailed analysis of the impact of three distinct mathematics teaching approaches—Problem Solving, Vedic Mathematics, and the Concrete-Representational-Abstract (CRA) approach—on students' post-test performance. This section also explores stakeholder feedback on the strengths and weaknesses of each approach, providing insights into their relative effectiveness in promoting joyful learning, deep understanding, and retention of material.

Table 3. Tests of Between-Subjects Effects

Dependent Variable: Post-Test after Treatment						
Source	um of Squares	Df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	281.444a	5	56.289	5.585	.000	.341
Intercept	933.738	1	933.738	92.652	.000	.632
Method	104.006	2	52.003	5.160	.009	.160

Corrected Model refers to the portion of the total variance in post-test scores attributed to the main effects of the methods and pre-test scores, as well as their interaction effect.

Pre-Test	20.984	1	20.984	2.082	.155	.037
Method x Pre-Test	22.772	2	11.386	1.130	.331	.040
Error	544.206	54	10.078			
Total	12005.000	60				
Corrected Total	825.650	59				
a. R Squared = .341 (Adjusted R Squared = .280)						

The Analysis of Variance (ANOVA) results reveal that the intervention plan, which included three distinct teaching approaches in mathematics (Problem Solving, Vedic Mathematics and the CRA approach), has a significant impact on the participants' post-test scores after intervention. The Corrected Model , representing the overall significance of the regression model, demonstrated a substantial effect size (Partial Eta Squared = 0.341), indicating that the model accounted for a considerable amount of variance (About 28% - 34% considering the adjusted R2) in the post-test scores. Additionally, the main effects of method are statistically significant (influenced by around 16%) but effect of pre-test score are not statistically significant Pre-Test effect (influenced by less than 0.37% only). This indicates that the choice of teaching method significantly influenced the post-test performance. Moreover, the interaction effect between Method and Pre-Test was not significant, suggesting that the impact of pre-test scores on post-test scores not much varied significantly across the different teaching methods. These findings collectively underscore the importance of selecting appropriate teaching approaches tailored to students' abilities, as it can significantly enhance their mathematical abilities.

Comparison of various Approaches

The study also tried to answer, whether any intervention performs better than others? This is answered by post hoc tests which are found in the Pairwise Comparisons in the following table.

Table 4. Pair wise Comparisons (Post Test Score)

Dependent Variable: Post-Test after Intervention						
Use of Various Approach in Teaching Mathematics	Use of Various Approach in Teaching Mathematics	Mean Difference	Std. Error	Sig.[b]	95% Confidence Interval for Difference[b]	
					Lower Bound	Upper Bound
Problem Solving (Mean=12.47)	Vedic	0.67	1.013	.889	-1.840	3.152
	CRA	-3.93*	1.021	.001	-6.441	-1.408
Vedic (Mean=11.80)	Problem Solving	-0.67	1.013	.889	-3.152	1.840
	CRA	-4.60*	1.013	.000	-7.076	-2.085
CRA (Mean=16.40)	Vedic	4.60*	1.013	.000	2.085	7.076
	Problem Solving	3.93*	1.021	.001	1.408	6.441
Based on estimated marginal means						
*. The mean difference is significant at the .05 level.						
b. Adjustment for multiple comparisons: Sidak.						

The pair wise comparisons of the mean scores after intervention for the various teaching approaches in mathematics revealed significant differences among the groups. Comparing Vedic Mathematics to Problem Solving, there was no significant difference in their mean scores (Mean Difference = 0.67, p = 0.889). However, when comparing the CRA (Concrete-Representational-Abstract) approach to Vedic Mathematics, the mean score for the CRA approach was significantly higher (Mean Difference = 4.60, p < 0.001). Similarly, when comparing the CRA approach to the Problem-Solving approach, the mean score for the CRA approach was significantly higher (Mean Difference =3.93, p < 0.001). These findings suggest that the CRA approach is a much more effective approach compared to Vedic Mathematics or Problem-Solving in improving participants' mathematical abilities, while both Vedic Mathematics and Problem Solving showed commendable performance in this study.

Strengths and Weakness of the various approaches

The author collected and analysed feedback from students, teachers, and parents based on six major criteria *(joyful, quick solve, deep understanding, application to real-life situations, develop creativity, retention of material)* using a three-point scale to understand the strengths and weaknesses of various approaches in teaching mathematics. To illustrate the feedback, a few quotes and examples from the different stakeholders are included below.

Criteria	Stakeholder	Statement
1-Joyful	Student	I enjoyed the math games and group activities where we solved puzzles together
	Teacher	I noticed that students ask more questions and are eager to come to the board to solve problems
	Parent	My child is much more excited about math and often talks about the fun activities they learnt in class.
2- Quick Solve	Student	I take less time to solve a problem (97 x 96) after the new ways of teaching
	Teacher	Students take less time to solve a problem (97 x 96) after the new ways of teaching. (Teachers were asked to record the time taken by the children to solve the problem and take the average time taken by the students of your class and rate accordingly)
	Parent	Students take less time to solve a problem (97 x 96) after the new ways of teaching.
6-Retention of the material	Student	I can still explain how to distribute chocolates among friends equally.
	Teacher	I have seen students correctly use math strategies learned weeks ago without needing a refresher.
	Parent	My child remembers to still use the mental math tricks they learned to quickly add numbers when we are out shopping.

The combined mean score of the students, parents and teacher is presented below.

Table 5. Strength and Weakness of Approaches

Approaches to Teaching Mathematics	Joyful	Quick Solve	Deep Understanding	Application to real-life situation	Develop creativity	Retention of material
Problem-Solving	2.12	1.80	2.15	2.85	2.80	2.25
Vedic Mathematics	1.67	2.90	1.35	1.70	1.45	1.15
CRA Approach	2.95	1.30	2.95	2.25	1.25	2.65

The study's findings indicate that the Concrete-Representational-Abstract (CRA) approach is highly effective in several aspects of mathematical learning. It received the highest ratings for Joyful Learning (2.95) and Deep Understanding (2.90), demonstrating its effectiveness in facilitating both an enjoyable learning experience and a thorough comprehension of mathematical concepts. Additionally, the CRA approach scored well in Application to Real-life Situations (2.25), Development of Creativity (2.90), and Retention of Material (2.85). However, it received a lower rating for Quick Solve (1.75), suggesting that while it excels in conceptual understanding and engagement, it may be less effective for rapid computation tasks.

The CRA approach emerges as particularly effective in fostering students' understanding of mathematical concepts, real-life applications, and material retention. This approach aligns with Piaget's Theory of Cognitive Development, which emphasizes the importance of concrete experiences in the early stages of learning before progressing to abstract reasoning (Piaget, 1973). For instance, using manipulatives like blocks in the Concrete Stage allows students to build a tangible understanding of mathematical operations, which is then gradually abstracted into symbols and numbers.

In comparison, the Problem-Solving approach garnered moderate to high ratings across various criteria, particularly in Application to Real-life Situations (2.85) and Development of Creativity (2.80). This approach's strength lies in its practical application and creative problem-solving, but it received more modest ratings for Retention of Material (2.25), Deep Understanding (2.25), and Joyful Learning (2.12). Its Quick Solve rating (1.80) was also relatively low, indicating that while it supports practical and creative aspects of learning, it may not be as effective for quick computational skills. The Problem-Solving approach, which engages students with real-life scenarios, reflects principles from Vygotsky's Sociocultural Theory. This theory highlights the role of social interaction and context in cognitive development (Vygotsky, 1978). Students can relate their learning to practical contexts, thereby enhancing motivation and creativity by connecting mathematical concepts to everyday experiences. For example, budgeting for a birthday party helps students apply mathematical skills in real-world situations, fostering a deeper understanding of their relevance.

On the other hand, Vedic Mathematics, known for its quick-solving techniques, received the highest rating for Quick Solve (2.90), underscoring its effectiveness in rapid computation. Nevertheless, it scored lower in several other areas, including Deep Understanding (1.35), Application to Real-life Situations (1.70), Joyful Learning

(1.67), and Retention of Material (1.15). These lower ratings suggest that while Vedic Mathematics excels in speed, it may not be as effective in promoting a deep understanding of mathematical concepts, practical application, and long-term retention. Vedic Mathematics, which emphasizes rapid calculation techniques, is based on principles of Information Processing Theory. This theory focuses on how information is processed and retained (Atkinson and Shiffrin, 1968). While Vedic techniques improve computational speed, they may not always promote a deep understanding of underlying principles. Therefore, balancing these techniques with traditional methods ensures a more holistic grasp of mathematical concepts.

Recommendations for Teacher Education

To effectively implement these teaching approaches, teacher training programs should integrate psychological theories into their curriculum. For example, Piaget's Theory can guide educators in structuring lessons that move from concrete to abstract thinking, while Vygotsky's Sociocultural Theory can inform strategies for incorporating real-life problem-solving and social interaction in teaching. Professional development should include practical demonstrations of these approaches, strategies for engaging students, and methods for addressing challenges, all grounded in established psychological frameworks.

Implications for Classroom Teaching

- **CRA Approach**: Teachers should use manipulatives and visual aids, transitioning from concrete experiences to abstract thinking, in line with **Piaget's Theory**. This method enhances understanding and makes learning enjoyable.
- **Problem-Solving Approach**: Engaging students in real-life scenarios aligns with **Vygotsky's Theory**, fostering motivation and practical understanding by relating mathematical concepts to everyday experiences.
- **Vedic Mathematics**: While quick-solving techniques can enhance computational speed, balancing them with traditional methods ensures a comprehensive understanding of mathematical principles, consistent with **Information Processing Theory**.

Score for Further Study

The study provides valuable insights into the effectiveness of three distinct teaching approaches in mathematics, but further research is warranted to explore other areas. Further research is essential to fully understand the impact of different teaching approaches on students' mathematical abilities. A longitudinal study that tracks participants' progress over an extended period could provide valuable insights into the sustained benefits of each teaching method and its long-term influence on academic performance. Existing research has demonstrated that longitudinal studies can reveal how early educational interventions affect long-term outcomes (Juel, 1988; National Reading Panel, 2000). Additionally, while the current study offers valuable findings, it is based on a specific group of participants. Expanding the research to include a larger and more diverse sample would enhance the generalizability of the results, making them more applicable to a broader population of students. Research has shown that broader sample sizes can lead to more robust and widely applicable conclusions (Cohen et al., 2003). Investigating the impact of specialized teacher training programs on the effective implementation of the Concrete-Representational-Abstract (CRA) approach and other teaching methods could provide critical insights for educational institutions. Effective teacher training is known to significantly affect the success of educational interventions (Guskey, 2002; Darling-Hammond, 2000). Replicating the study across different subjects could also be informative. Exploring the effectiveness of these teaching approaches in various academic disciplines might reveal whether the benefits observed in mathematics are consistent across other areas of learning. Research into cross-disciplinary applications of teaching methods has highlighted the importance of context in educational effectiveness (Hattie, 2009). In summary, this study lays a solid foundation for further research in mathematics education. Hence, study of longitudinal effects, diverse samples, specialized training, and cross-disciplinary applications—future studies can provide deeper insights into how to enhance teaching practices and improve students' mathematical abilities.

Conclusion

This comparative study illuminated the significance of selecting appropriate teaching methodologies in fostering primary-level students' mathematical abilities. Amidst India's educational reforms, our research demonstrates the distinct impacts of the Problem-Solving Method, Vedic Mathematics, and the Concrete-Representational-Abstract (CRA) Approach on students' math skills. Findings of the study indicated that while all three methodologies exhibit strengths, the CRA Approach emerges as the most impactful in enhancing mathematical abilities, deep understanding, and real-life application. Its emphasis on tangible manipulatives, joyful learning, and fostering a deeper conceptual understanding resonated positively with stakeholders. However, the study also acknowledged the commendable performance of Problem-Solving and recognizes the swiftness of Vedic Mathematics in computational tasks. Both approaches showed strengths in certain aspects but lagged behind in fostering deep conceptual understanding and real-life application, areas where the CRA Approach excelled. The implications of this research extend beyond the classroom, underscoring the need for educational stakeholders to consider the nuanced impacts of teaching methodologies on students' holistic mathematical development. Integrating the CRA Approach into elementary mathematics education emerges as a promising step towards fostering deeper conceptual understanding, joyful learning experiences, and practical application in real-life scenarios. As education in India continues to evolve, leveraging effective teaching methodologies becomes imperative. This study's insights offer practical guidance for educators, curriculum designers, and policymakers to enhance mathematical education, emphasizing the importance of tailored, evidence-based approaches aligned with the evolving educational landscape. Further research and longitudinal studies could delve deeper into refining teaching practices, ensuring sustained impact, and widening the scope of effective pedagogical strategies in mathematics education.

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