Diagnostic Competence of Pre-service Mathematics Teachers: Unpacking a Complex Construct

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Abstract

Despite efforts for equity and access to mathematics education for all, the number of struggling students is trapped in a vicious circle of unsatisfactory performance to achieve the learning outcomes in mathematics. Nevertheless, against the backdrop of the large-scale heterogeneity of students in Indian classrooms, the diagnostic competence of mathematics teachers plays a significant role in students' mathematics learning. Ergo, the present study aimed to examine the level of diagnostic competence of pre-service mathematics teachers. Descriptive survey method was used to conduct the study. Forty-two final year pre-service mathematics teachers participated in the study. A self-developed tool based on the Perception-Interpretation- Decision-Making (P-I-D) triad model (Blomeke et al., 2015) was administered to the selected sample. Non-parametric statistics χ^2 *test was used to analyse the data. The findings reveal a low level of diagnostic competence of pre-service mathematics teachers along with dimensions (detecting, describing, and remediating). Further, according to the findings, very few pre-service mathematics teachers have L-5 diagnostic competence i.e., providing remediation specifically after describing the error pattern in students' response accurately and exploring the possible reasons. Furthermore, gender difference was not found in the diagnostic competence of pre-service mathematics teachers. The findings of the study highlight the pressing need to develop a support system for enhancing the diagnostic competence of pre-service mathematics teachers to ensure early diagnosis and remedial support for the struggling students. Implications and directions for further research are outlined.*

Keywords: *Diagnostic competence, pre-service mathematics teachers, struggling students, mathematics learning*

1. Introduction

Due to ever-changing, and increasingly complex challenges faced by human beings, there is a pressing need to educate the future generation as creative citizens to achieve the ideals of peace, freedom, and social justice that leads to the welfare of human beings globally. For this, mathematics provides a suitable platform. National Education Policy (NEP 2020) highlights that mathematics and mathematical thinking are very important in the numerous upcoming fields and professions which involve artificial intelligence, machine learning, coding, data science, etc. It plays a pivotal role in the everyday life of human beings as well as occupies an important place in the school curriculum. It opens the door for the prosperity of any nation and helps in providing solutions to the problems posed in social, cultural, and natural environments. In this changing world, those who understand and can-do mathematics will have significantly enhanced opportunities and options for shaping their futures (NCTM, 2000). National Council of Teachers of Mathematics (NCTM, 2000) highlights that all students deserve to learn mathematics and have an equal right to quality mathematics education. In support, the National Focus Group on Teaching Mathematics (NFGTM, 2006) advocates that all students can learn mathematics and that all students need to learn mathematics. Therefore, to achieve the target to ensure quality education, National Education Policy (NEP 2020) recommends transforming the assessment system in education for shifting from testing rote memorisation to competency-based learning. Ergo, the Ministry of Education, India has introduced 'Structured Assessment for Analyzing Learning levels' (SAFAL) a competency-based assessment to ensure the students' progress on foundational skills and learning outcomes by providing diagnostic information about students' learning to teachers, schools, and their parents (NEP 2020). But despite efforts for inclusive and equitable education for all, the number of students nationally as well as globally are struggling from their unsatisfactory performance in mathematics learning (NAS, 2021, 2017; ASER, 2022, 2018; & OECD, 2016).

Instruction and assessment both are integral parts of the teaching-learning process. Assessment of student learning outcomes is an important key aspect for improving the quality of instruction at the entire spectrum of education. The assessment process is not only helpful for students to know their progress but also helpful for teachers and teacher educators to assess the effectiveness of their pedagogical practices. Hence, teachers are expected to identify, interpret and decision-making regarding improving the students' learning through error analysis and accordingly improving their instruction quality. These abilities to analyse students' errors and understand student thinking at every stage during solving problems in mathematics have been introduced in terms of the concept of diagnostic competence (Aufschnaiter et al., 2011).

1.1 Diagnostic Competence

Diagnostic competences, a relevant but complex construct (Leuders et al., 2020) is an essential facet of teacher competence (Wildgans-Lang et al., 2020). It consists of different diagnostic activities like gathering and interpretation of information on the students' learning condition, learning process and learning outcomes that is determined by formal testing, observation, students' writings, interview with students etc. Ergo, the teachers' knowledge, skills, motivation and beliefs relevant to the diagnostic activities is summarised in term of diagnostic competence (Leuders et al., 2020; & Aufschnaiter et al., 2015). It is associated with teachers' skills in understanding and analysing students' thinking process —without any concern for grading them (Prediger, 2010) and for identifying and correcting specific error patterns students exhibit in their work to promote further learning (Larrain & Kaiser, 2019). From this perspective, it gains importance in two respects namely first, as an opportunity to diagnose learning difficulties and to create awareness and support for the performance and understanding of individual students. Second, it seems to be a remarkable starting point for research in the field of mathematics teaching-learning process. Diagnostic competence is not separately discussed in the different models of professional competence of teachers (COACTIV). It is a sub facet of Pedagogical Knowledge (PK) and Pedagogical Content Knowledge (PCK). To plan further support individually or in a group and to inform students and their parents and decide the procedures, valid knowledge about learners is required by using diagnostic activities. These activities like teachers' knowledge, skills, motivations, and beliefs are summarised as diagnostic competencies

(Aufschnaiter et al., 2011; Herppich et al., 2018). Diagnostic competence is used for conceptualizing a teacher's competence to analyse and understand student thinking and learning processes without immediately assessing them. According to Heitzmann et al. (2019), it can be defined as "individual dispositions enabling people to apply their knowledge in diagnostic activities according to professional standards to collect and interpret data to make decisions of high quality". It has been demonstrated by research studies (Hoth et al, 2016) that school teachers who have the competence to diagnose students in mathematics are not necessarily effective at diagnosing them in writing or reading. In addition, a number of research studies examined the relationship between instructional quality implemented by mathematics teachers' and their competence (Blomeke, Kaiser, Koni, & Jentsch, 2020; & Kunter et al., 2013). Therefore, there is a pressing need to adopt a domain-specific approach for exploring the diagnostic competencies of pre-service mathematics teachers.

1.2 Diagnostic Process

Several research studies have investigated the quality of teachers' judgements of students' competences. However, ample evidence has not been found about the processes that lead to these judgments and ways to promote the processes in the early phase of teacher training (Wildgans-Lang et al., 2020). It is assessed by examining teachers' ability to analyse and identify errors in the students' work, anticipate common errors, and estimate the difficulty level of given tasks in order (Ostermann et al., 2018). It has four different components namely understanding skills include consideration and scrutiny; analytical skills include knowing/implementation, and interpretation (Prediger, 2010). In addition, Fischer et al., (2014) define eight diagnostic activities for scientific reasoning processes in diagnostic processes namely- (i) problem identification (ii) asking questions (iii) generating hypotheses (iv) construct artifacts (v) generating evidence (vi) evaluating evidence (viii) drawing conclusions, and (viii) communicate results. The explicitly model process of perception (P), interpretation (I), and decision-making (D), such as the concept of noticing (Santagata & Yeh, 2016) and of teacher decision-making are mostly used to investigate or to systematically influence teacher behavior during the instruction process. Blömeke et al. (2015) propose a model of competence as a continuum (Figure 1), which embodies both perspectives and also includes cognitive processes that leads to observable behavior (as cited in Leuders et al., 2018).

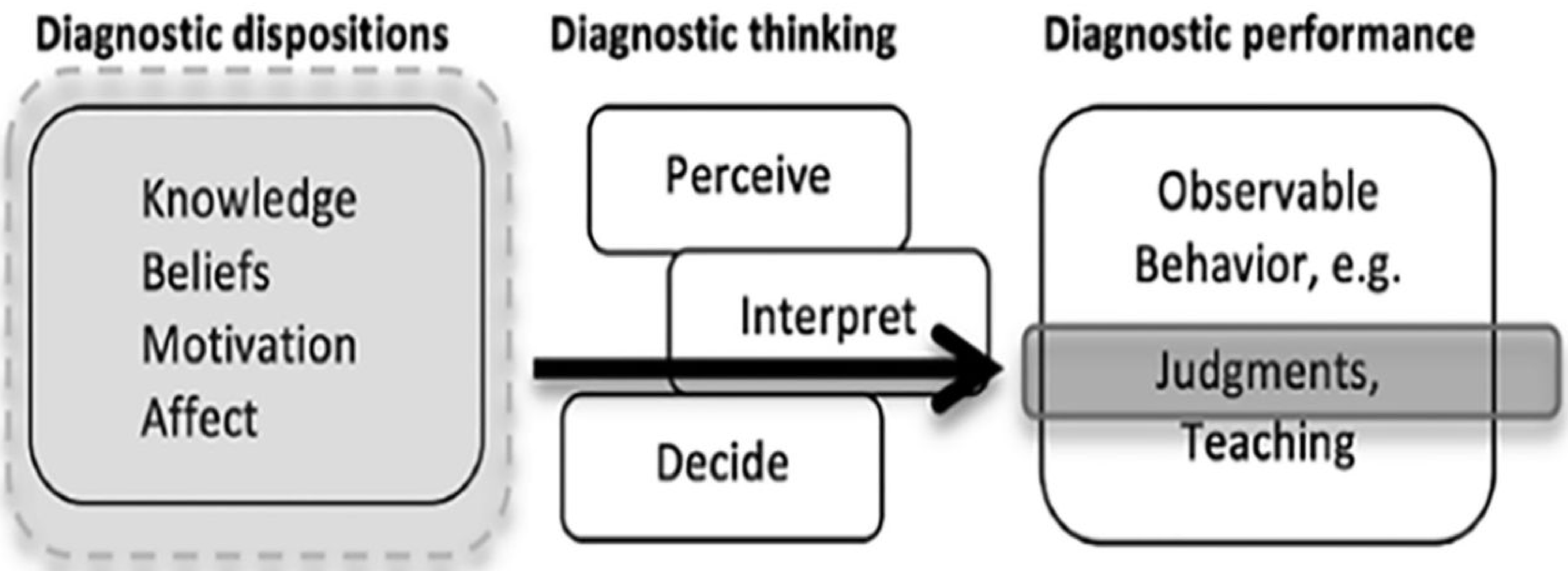

Figure 1: Perspective of Diagnostic Competence

Diagnostic Dispositions comprise knowledge, beliefs, motivational and affective factors that are relatively stable within a person, and which contribute to the ability to act successfully in diagnostic situations. Diagnostic Thinking can be regarded as a set of situation-specific cognitive functions or processes of perception, interpretation, and decision-making. Diagnostic Performance relates to observable behavior in diagnostic situations as they arise in the professional life of a teacher. It is the result of interaction between diagnostic dispositions and diagnostic thinking. In the present study, to assess the diagnostic competence, the items related to the knowledge of students' thinking, task properties or ability testing have been considered.

2. Rationale of the Study

According to the School Education Quality Index (SEQI, 2019) report, Bihar has the average mathematics scores of Class VIII (45.0 %) which is a little higher than the average national score but comparatively lower than Rajasthan (57.0 %). In addition, the Annual Status of Education Report (ASER, 2018) shows a disheartening figure in the form of learning outcomes of Class VIII students especially in the context of rural middle schools of Bihar which indicates that 44 per cent of children in Class VIII can solve a division of 3-digit by 1-digit number correctly. In addition, the World Development Report (2018) also brings attention that in Bihar state, only 10.5 per cent of tested public-school teachers can solve a division of three-digit by one-digit and by showing the steps correctly. In the National Achievement Survey (2017) based on Learning Outcomes, the percentage of correct responses (on an average) for Class VIII in Mathematics at the National level is 42 per cent. Furthermore, gender differences do appear in students' mathematics performance and favoring boys' students (NAS, 2021). Moreover, the mathematics performance of students in NAS (2021) is declined from NAS (2017). Furthermore, ASER report (2022) highlights the very weak performance of students in mathematics learning. However, other factors may be responsible for this unsatisfactory performance but one of the factors is students' learning during the COVID 19 pandemic. Therefore, post pandemic, diagnostic competence of mathematics teachers will be helpful to recover the learning loss of students. Hernadi, Ekayanti, and Jumadi (2020) reported the main obstacle in mathematics learning at the junior high school level is the lack of students' skills in performing basic operations of integers and fractions. Without proper knowledge of arithmetic skills, students face many difficulties learning any mathematical topics, e.g., 'algebraic forms' involving variables as abstractions of numbers. During the pilot study, the investigator found that most of the middle school students are unable to answer the question: what is the square root of 0.9? Besides this, the majority of pre-service mathematics teachers are also unable to answer why the square roots of 0.9 are greater than 0.9 whereas the square roots of 9 are less than 9. Hence, it is a great need to take initiatives for fostering the diagnostic competence of pre-service mathematics teachers in terms of identifying the error pattern based on the students' response, cause of low-performance, and identifying different factors affecting their performance negatively. Several research studies have already shown the evidence in terms of the importance of diagnostic competence of mathematics teachers on students' performance (Hoth et al., 2016; Kaplan & Argun, 2017; Klug et al., 2013; McNeil, 2021; Guruzhapov et al., 2019). Ostermann et al. (2018) conducted a study to assess teachers' diagnostic competence by examining teachers' ability to analyse and identify errors in the students' work, anticipate common errors, and estimate the difficulty level of given tasks in order. Against a large-scale heterogeneity of students in Indian classrooms, the diagnostic competence of teachers plays a pivotal role to provide a remedial intervention for those who may be at-risk for struggling to overcome the situation. Despite the importance of diagnostic competence of teachers in T-L process, ample evidence of research studies in mathematics education has not been found in India especially in Bihar State of India as reviewed by the investigator.

Therefore, in the light of above discussion, the present study has been conducted to address the following research questions:

1. What is the level of diagnostic competence of pre-service mathematics teachers?
2. Do gender differences exist in the diagnostic competence of pre-service mathematics teachers?

3. Methodology

3.1. Method

According to the nature of the study, the descriptive survey method was used to conduct the study.

3.2. Participants

The participants for the study were final year students of Bachelor of Science Education programme (B.Ed. & B.Sc. B.Ed.) known as pre-service mathematics teachers from Bihar State of India considered as the population of the study.

Ss of the study consisted of forty-two pre-service mathematics teachers (30 male pre-service teachers & 12 female pre-service teachers) from the teacher training institutions located in Bihar State of India were conveniently selected and voluntarily participated in face-to-face mode during COVID 19 pandemic. They had completed the pedagogy and micro-teaching course in mathematics education as part of the curriculum of their teacher education programme.

3.3. Instrument

A self-developed instrument (paper-and-pencil test) based on situation-based items from the arithmetic branch of mathematics at the middle level was used to assess the diagnostic competence of prospective mathematics teachers. In the present study, the diagnostic competence of pre-service mathematics teachers has operationally defined the ability for identifying, interpreting, and decision making in diagnosing the error pattern in arithmetic. Hence, the diagnostic competence of pre-service mathematics teachers has been represented by the composite scores obtained on identifying (detecting/perception), interpretation (describing error pattern) remediating/decision making (basic & specific) dimensions of the Mathematics Diagnostic Competence Test (as shown in Table 1) which was developed by the investigator based on the P-I-D triad model (Blomeke et al., 2015).

Table 1

Diagnostic Competence Test

Level	Dimension	Description
Level-5	Remediating-Advanced (How to reconcile the error pattern to the correct and relevant conceptual and procedural knowledge-Advanced)	Use manipulatives or visual models to explain the procedure of determining $\frac{1}{3}$ of something. For example: $\frac{1}{3}$ of 6 is 2, and $\frac{1}{3}$ of 15 is 5. Use of operation for multiplication: $\frac{1}{3}\times 6=2$, and $\frac{1}{3}\times 15=5$. Similarly, illustrate $\frac{1}{3}$ and $1\frac{1}{3}$ of whole numbers, draw a model showing $\frac{1}{3}$ and $1\frac{1}{3}$, and shade one-third and $1\frac{1}{3}$ of something. Thereafter, make the connection with the procedure of multiplication like double, triple and n times expressions and elaborate how one-third of $1\frac{1}{3}$ can't be 1.

Level-4	Remediating-Basic (How to reconcile the error pattern to the correct and relevant knowledge-Basic)	Describing the error pattern, The word 'of' in math is used as a keyword for 'multiplication'. Taking 'x' of 'y' would mean to multiply x×y, and not to subtract 'x' from 'y'. Providing help to understand how multiply or divide is not clear from the provided response of participants.
Level-3	Describing- Detailed (Describes error pattern accurately and explore possible reasons)	Identifying and describing error patterns comprehensively Understanding how to use 'of' in mathematics, therefore, used subtraction instead of multiplication, memory deficit in respect of mastery on factual information.
Level-2	Describing- Basic (Describes error pattern accurately, but in general terms)	Identifying error pattern and describing in general form for example only gets rid of the fraction.
Level-1	Detecting (Detects the presence or absence of an error pattern)	Identifying error pattern either by effort or by guess
Level-0	In-evident (Fails to detect the presence or absence of an error pattern)	Not identifying error pattern Providing either irrelevant or inaccurate response

3.4. Reliability and Validity

The appropriateness of the items of the tool in terms of trustworthiness and truthfulness for assessing diagnostic competence of pre-service mathematics teachers were piloted and checked by two subject experts and one educational psychologist. They reviewed all items of the tool by considering the fundamental aspects like content, items, language, vagueness, length, dimensions, etc. After receiving the experts' feedback, a minor revision was applied to the tool for improving its validity.

3.5. Research procedure and data collection

Due to the rural and urban background of the participants, the instructions and items of the diagnostic competence test were translated from English to Hindi, with the back-translation procedure to ensure accuracy and equivalency. Before administration of the test on a selected sample, the consent from the school authority has been taken with all ethical considerations and following the precautionary measures of COVID 19. In face-to-face mode, data from the pre-service mathematics teachers was collected by using a self-developed paper-pencil mathematics diagnostic competence test.

3.6. Data analysis

Data analysis in this study involved selecting, sorting, categorising, synthesising, summarising, and interpreting data based on the five dimensions of diagnostic competence test namely- ability to detect the error, describe error pattern in general, describe error pattern in specific, remediation in general and specific. As per the nature of data, Chi-square test, and percentage analysis were used to analyse the data.

4. Result

The mean, standard deviation, skewness, and kurtosis on the diagnostic competence test are presented and summarised in Table 1.

Table - 1

Descriptive Statistics on Diagnostic Test (N=42)

Variable	Minimum	Maximum	Mean	SD	Kurtosis	Skewness
Diagnostic Competence	19	1	10.71	4.62	-1.11	-0.16

Tables 1 and 2 show the basic statistics of the group of participants selected in the study. It is apparent from Table 1 that the SD value of diagnostic competence is 4.62 for a mean of 10.71 Which is very high and approximately equal to one and a half to mean. It shows that the group is very heterogeneous and does not satisfy the assumptions of parametric statistics.

Table 2

Details of the Participants (N=42)

Gender	Diagnostic Competence of Mathematics Teachers					Total
	Detecting	Describing		Remediating		
	D-1	D-2	D-3	D-4	D-5	
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	
Female (N=12)	1.66 (1.49)	1.5 (1.59)	2.56 (1.52)	1.56 (1.54)	2.60 (1.27)	9.9 (3.94)
Male (N=30)	1.75 (1.13)	1.91 (1.31)	3.58 (1.08)	2.66 (2.01)	2.83 (1.85)	12.75 (5.69)
Total	1.69 (1.38)	1.61 (1.51)	2.85 (1.47)	1.88 (1.74)	2.66 (1.44)	10.71 (4.62)

It can be seen from the statistics values shown in Table 1 and Table 2 that due to the small sample size and skewed nature of data collected from diagnostic competence test, the non-parametric statistical techniques were found appropriate to analyze the data. Ergo, according to the nature of data, percentage, and $x2$ test were used to analyze the data.

Item 1 [Addition]

In the Mathematics classroom, a student solved the four problems of addition given by mathematics teachers in the following manner:

38	45	32	30
9	37	10	49
+ 65	+ 29	+ 19	+ 5
112	102	61	111
A	B	C	D

Based on the response patterns of that student for the solutions of the above-mentioned math problems, answer the following questions:

1. Which one of the following is correct?
   (a) A & C are correct.
   (b) B & D are incorrect.
   (c) A, B, C, D all are incorrect.
   (d) A & C are correct but B & D are incorrect
2. If the solution is not correct, explain/describe the students' error pattern.
3. Being a teacher, how will you help the student to learn the procedure of addition of numbers?

Table 3

Diagnostic Competence of pre-service Mathematics Teachers [Addition]

S.No.	Gender	Detecting Error Pattern F (%) (1)	Describing in General (%) (2)	Describing in Specific (%) (3)	Remediating in General (%) (4)	Remediating in Specific (%) (5)
Item No 1	Female	0.00	21.4	8.53	21.4	8.53
	Male	3.33	38.0	23.33	63.33	23.33
	Total	2.38	59.52	19.00	66.66	19.00
	χ^2	0.23	0.89	0.47	0.13	0.47
	Significance	NS	NS	NS	NS	NS

S-Significant (p 0.05)
NS-Not Significant (p 0.05)

As can be seen from Table 3 that only 3.33 per cent of pre-service male mathematics teachers detected the presence of error based on the students' response for item no 1. However, female pre-service mathematics teachers failed to detect the presence of error pattern based on the student's responses. Further Table 3 shows that 19.00 per cent of pre-service mathematics teachers identified and described the error pattern in general. Furthermore, Table 3 depicts that after identifying and describing the error pattern, 66.66 per cent of pre-service mathematics teachers provided the remediation in general whereas only 19.00 per cent of teachers provided systematically. Moreover, only one female pre-service teacher described the error pattern with systematically. It indicates that only 19.00 per cent of pre-service mathematics teachers were found to be better in respect of the diagnostic competence. It is evident from Table 3 that the obtained $\chi 2$ values [$\chi 2$ = 0.23, 0.89, 0.47, 0.13 & 0.47; df=1; p > 0.05] were not found to be significant on pre-service mathematics teachers' diagnostic competence with respect to diagnostic competence. It indicates that there is no significant difference between the diagnostic competence of male and female pre-service mathematics teachers. Therefore, it reveals that the gender difference does not appear in the diagnostic competence of the pre-service mathematics teachers.

Item 2 [Square Roots]

<table>
<tr><td colspan="3">A mathematics teacher gave a mathematical problem given in column-A. The four students of the class solved the same problem</td></tr>
<tr><td>Column – A (Problems)</td><td colspan="2">Column – B (Solution)</td></tr>
<tr><td rowspan="4">Find out the square root of 0.9.</td><td>Student A</td><td>a. 81</td></tr>
<tr><td>Student B</td><td>b. $(.9)^{0.5}$</td></tr>
<tr><td>Student C</td><td>c. 0.3</td></tr>
<tr><td>Student D</td><td>d. 0.94</td></tr>
<tr><td colspan="3">Based on the response patterns of students (A, B, C & D) for the solution of the math problems, answer the following questions:
1. Which one of the solutions of students is correct-?
(i) Only C (ii) Only D (iii) Only A (iv) None of the above
2. Explain the error pattern to the wrong solution to the problem
3. Being a teacher, how will you help the students to find out the square root of the decimal number?</td></tr>
</table>

Table 4

Diagnostic Competence of Pre-service Mathematics Teachers [Square Roots]

S.N.	Gender	Detecting Error Pattern F (%)	Describing in General (%)	Describing in Specific (%)	Remediating in General (%)	Remediating in Specific (%)
		(1)	(2)	(3)	(4)	(5)
Item No 2	Female	83.33	41.66	4.76	33.33	0.00
	Male	56.66	43.33	13.33	30.00	6.66
	Total	64.88	45.23	14.28	30.95	4.76
	χ^2	1.62	0.08	0.43	0.02	0.01
	Significance	NS	NS	NS	NS	NS

S-Significant (p 0.05)

NS-Not Significant (p 0.05)

The statistical values shown in Table 4 reveal that only 64.88 per cent of pre-service mathematics teachers detected error based on the student's response for item no 2. The percentage of female pre-service teachers (45.23%) is higher in respect of describing error patterns in general than describing in specific (14.28%). Further, it can be seen from Table 4 that after describing the error pattern, only 30.95 per cent of pre-service teachers mentioned the remediating process for the students in general. But only 4.76 per cent of pre-service mathematics teachers provided the remediation for the students in an appropriate manner. However, no female pre-service teachers provided the remedial intervention in specific. It indicates that only 4.76 per cent of pre-service mathematics teachers were found to be better at their diagnostic competence. Furthermore, it is evident from Table 4 that the obtained $\chi 2$ values [$\chi 2$ = 1.62, 0.08, 0.43, 0.02 & 0.01; $df=1$; $p > 0.05$] were not found to be significant on pre-service mathematics teachers' diagnostic competence with respect to diagnostic competence. It indicates that there is no significant difference between the diagnostic competence of male and female pre-service mathematics teachers. Therefore, it reveals that the gender difference does not appear in the diagnostic competence of the pre-service mathematics teachers.

Item 3 [Place of Decimal Values]

Suppose a mathematics teacher gave a mathematics problem to students that if we subtract decimal four from decimal nine four. Thereafter, a few students did the solution in the following manner.	.94 − . 90
By observing the response of students for the solution of the problem, answer the following questions: 1. Is the correct solution given by the student? (Y/N) 2. If NOT then Describe/explain the students' error patterns. 3. Being a teacher how will you help students to learn the subtraction of decimal numbers.	

Table 5

Diagnostic Competence of Pre-service Mathematics Teachers [Place of Decimal Values]

S.N.	Gender	Detecting Error Pattern F (%)	Describing in General (%)	Describing in Specific (%)	Remediating in General (%)	Remediating in Specific (%)
		(1)	(2)	(3)	(4)	(5)
Item No 3	Female	100	100	58.33	75	16.66
	Male	86.66	83.33	26.66	46.66	16.66
	Total	90.47	88.09	35.71	54.76	16.66
	χ^2	0.56	0.95	2.49	1.75	1.47
	Significance	NS	NS	NS	NS	NS

S-Significant (p 0.05)
NS-Not Significant (p 0.05)

As can be seen from Table 5 that only 90.47 per cent of pre-service mathematics teachers detected the presence of error based on the students' response for item no 3 and 88.09 per cent of pre-service mathematics teachers identified and described the error pattern in general. Whereas only 35.71 per cent of pre-service mathematics teachers described the error pattern in specific. 54.76 per cent of pre-service mathematics teachers provided the remediation for the students in general and only provided remediation in specific (16.66%). It indicates that only 16.66 per cent of pre-service mathematics teachers were found to be better at their diagnostic competence. Furthermore, it is evident from Table 5 that the obtained $\chi2$ values [$\chi2$ = 0.56, 0.95, 2.49, 1.75 & 1.47; df=1; p > 0.05] were not found to be significant on pre-service mathematics teachers' diagnostic competence with respect to diagnostic competence. Therefore, it reveals that the gender difference does not appear in the diagnostic competence of the pre-service mathematics teachers.

Item 4 [Fractions]

In mathematics classroom, if a student solves the problems as given in column B of the fractions given in column A:	
Column – A (Problems)	Column – B (Solution)
Find out $\frac{1}{3}$ of $1\frac{1}{3}$	$1\frac{1}{3} - \frac{1}{3} = 1$
Find out $\frac{1}{2}$ of $2\frac{3}{5}$	$2\frac{3}{5} - \frac{1}{2} = 2\frac{2}{3}$
Find out $\frac{3}{7}$ of $3\frac{1}{7}$	$3\frac{1}{7} - \frac{3}{7} = 3\frac{2}{7}$
By observing the response pattern of students for the solutions of the problems, answer the following questions: Which one of the following solutions is correct-? (i) Only A (ii) B & C (iii) A, B & C (iv) None of the above If the solution is not correct, describe/explain the student's error pattern. Being a teacher how will you help students to solve such type of problem (anyone)	

Table 6

Diagnostic Competence of Pre-Service Mathematics Teachers [Fractions]

S.No.	Gender	Detecting Error Pattern F (%)	Describing in General (%)	Describing in Specific (%)	Remediating in General (%)	Remediating in Specific (%)
		(1)	(2)	(3)	(4)	(5)
Item No 4	Female	75.00	66.66	41.66	66.66	25.00
	Male	56.66	56.66	10.00	33.33	3.00
	Total	61.90	59.52	19.04	42.85	9.52
	χ^2 Value	0.56	0.62	5.36	2.64	2.49
	Significance	NS	NS	NS	NS	NS

S-Significant (p 0.05)
NS-Not Significant (p 0.05)

It is evident from Table 6 that only 61.90 per cent of pre-service mathematics teachers detected the presence of error based on the students' response for item no 4. Table 6 depicts that 59.52 per cent and 19.04 per cent of pre-service mathematics teachers identified and described the error pattern in general and in specific respectively. Further, Table 6 shows that only 42.85 per cent of pre-service mathematics teachers provided the remediation for the students in general but in specific only by 9.42 per cent of pre-service mathematics teachers. Furthermore, statistical values shown in Table 6 indicates that the obtained $\chi2$ values [$\chi2$ = 0.56, 0.62, 5.36, 2.64 & 2.49; df=1; p > 0.05] were not found to be significant on pre-service mathematics teachers' diagnostic competence with respect to diagnostic competence. It indicates that there is no significant difference between the diagnostic competence of male and female pre-service mathematics teachers. Therefore, it reveals that the gender difference does not appear in the diagnostic competence of the pre-service mathematics teachers.

Item 5 [BODMAS]

A mathematics teacher gave the mathematical problems given in Column A. A student of the class solved the problems given in column-B in the following manner

Column – A (Problem)	Column – B (Solution)
(i) $1 + 1 - 1 \times 1 \div 1^0$	(a) $2 - 1 = 1$
(ii) $2 + 2 - 2 \times 2 \div 2^0$	(b) $4 - 2 = 2$
(iii) $3 + 3 - 3 \times 3 \div 3^0$	(c) $6 - 3 = 3$
(iv) $4 + 4 - 4 \times 4 \div 4^0$	(d) $8 - 4 = 4$

By observing the response patterns of students for the solution of the problems, answer the following questions:

1. Which one of the following is correct?
   (a) Only A
   (b) A & C
   (c) A, B, C & D
   (d) None of the above
2. Explain the error pattern to the wrong solutions to the problem
3. Being a teacher how will you help students for solving such type of problem (anyone)

Table 7

Diagnostic Competence of Pre-Service Mathematics Teachers' [BODMAS]

S.No.	Gender	Detecting Error Pattern F (%)	Describing in General (%)	Describing in Specific (%)	Remediating in General (%)	Remediating in Specific (%)
		(1)	(2)	(3)	(4)	(5)
Item No 4	Female	75.00	75.00	33.33	75.00	25.00
	Male	90.00	90.00	26.66	46.66	6.00
	Total	85.71	85.71	28.57	54.76	11.90
	χ^2 Value	0.58	0.58	0.00	1.75	1.27
	Significance	NS	NS	NS	NS	NS

S-Significant (p 0.05)
NS-Not Significant (p 0.05)

As statistical values shown in Table 7 that only 85.71 per cent of pre-service mathematics teachers detected the presence of error pattern and described error pattern based on the students' response for item no 5. Whereas 28.57 per cent of pre-service mathematics teachers described the error pattern in specific. Table 7 depicts that

54.76 per cent of pre-service mathematics teachers provided the remediation for the students in general whereas 11.90 per cent provided remediation in specific. Furthermore, it is evident from Table 7 that the obtained $\chi 2$ values [2 = 0.58, 0.58, 0.00, 1.75 & 1.27; df=1; p > 0.05] were not found to be significant on pre-service mathematics teachers' diagnostic competence with respect to diagnostic competence. It indicates that there is no significant difference between the diagnostic competence of male and female pre-service mathematics teachers. Therefore, it reveals that the gender difference does not appear in the diagnostic competence of the pre-service mathematics teachers.

Discussion

The findings of this study revealed that pre-service mathematics teachers have shown a low level of diagnostic competence in the arithmetic branch of mathematics. A low level of ability of pre-service mathematics teachers was found to detect the error pattern especially in the area of addition of numbers and square roots. However, most of the pre-service mathematics teachers were found to be able to detect the error pattern in general. But very few pre-service teachers were found to detect the error pattern in specific. Similar findings are also supported by different research studies (Liu, Jacobson & Bharaj, 2020). Hoth et al. (2016) noted that teachers who have limited knowledge frequently overlook the learning aspects in-depth and are more focused on student behavior. Further, most of the pre-service mathematics teachers mentioned the remedial procedure in general after detecting the error pattern. Whereas very few pre-service mathematics teachers were found to be better for providing remedial intervention in specific based on the pattern of students' response. For providing the remediation systematically, scholarly attention is needed not only based on students' mistakes, errors or by right/wrong attitudes but also with the perception of observing things through students' eyes (Prediger, 2010).

Furthermore, the findings of the study challenge the historical notion that males were thought to be more suited to teach mathematics than females (Leder, 2019; Hussain, Farooq & Mahmood, 2018). In contrast, in the present study, the gender difference was not found with respect to the diagnostic competence of pre-service mathematics teachers. It means that a greater male variability hypothesis and superiority in teaching competence in mathematics does not exist i.e., both have shown the similar extent of diagnostic competence of mathematics teachers. On the other hand, Hastedt et al. (2021) reported a low level of self-efficacy of female pre-service mathematics teachers and underestimate their capacities in transmitting mathematical knowledge to the students.

Conclusion

The findings of the study reveals that most of the pre-service mathematics teachers have shown a low level of diagnostic competence in mathematics learning. After analysis of the responses, it is revealed that the average number of the pre-service teachers described the error pattern in general and similarly provided the remediation in general. Whereas a very few pre-service mathematics teachers were found those who've provided the remediation systematically after describing error patterns systematically. Therefore, these issues can be kept in mind while providing the training for pre-service mathematics teachers like training for developing lesson plans and preparation for school internship. Further, it can be seen from the findings of the present study that gender difference does not appear in respect of diagnostic competence of pre-service mathematics teachers. On the other hand, the findings contradict earlier studies that female teachers may be attributed to the knowledge of students' thought processes, conceptions & misconceptions, and text analysis skill level (Otun, 2022) and course work performance (Isiksal, 2005).

Educational Implications of the Study

Due to a large scale of heterogeneity in Indian classrooms, teachers are required to individualise teaching strategies and provide targeted support to struggling students for promoting their mathematics learning. Thus, diagnosing learners' difficulty and weakness is one of teachers' most central tasks in the mathematics classroom. Hence, the findings of the present study will be helpful for pre-service mathematics teachers as well as school teachers, teacher educators, and curriculum developers as follows:

(i) Students' errors in mathematics learning are a rich source of evidence about students' mathematical ideas. Therefore, teachers' diagnostic competence plays a pivotal role in the judgment accuracy of teachers and are highly relevant for the quality of instruction and adaptive teaching (Ohle & McElvany, 2015) as per the diverse needs of learners.

(ii) Integrating diagnostic practices into mathematics pedagogy course during pre-service teachers training programme can enhance their formative assessment skills that facilitates how to diagnose the students' difficulties and weakness in their learning therefore, accordingly provide the remedial support for them as earliest. Furthermore, modification in curriculum may ensure early diagnosis and remedial support among pre-service mathematics teachers.

(iii) After identifying the level of diagnostic competence of pre-service teachers, a support system can be developed for teachers and pre-service teachers also. Diagnostic competence of pre-service mathematics teachers leads to more accurate diagnoses of students' difficulties in mathematics learning. Further it leads to overcoming the students' problem and enhancing students' performance in mathematics learning.

Limitations and Future Perspectives

Despite having the strengths of the methodological part, this study has some limitations by which the generalization of findings of this study may be less valid like 1) small sample size in extraordinary circumstances due to COVID-19 pandemic situation; 2) lack of triangulation method like classroom observations and interview procedure may use for capturing the clear picture of the diagnostic competencies of pre-service mathematics teachers; and 3) lack of establishing validity of tool related with other criterion. Because, assessment skills are context-specific (Blomeke et al., 2015), therefore, a video-based simulation and written notes (Wildgans-Lang et al., 2020) may provide better insight for assessing diagnostic competence of mathematics teachers. The present study is only limited to the arithmetic branch of mathematics, therefore, algebra, geometry, data handling, trigonometry etc. may be considered in the future research studies. Further research studies are needed to confirm the findings of the study as required to correlate the greater use of diagnostic competence of pre-service mathematics teachers and struggling students' performance in mathematics. Ergo, cross-lagged panel study (Tyagi & Singh, 2014) and experimental research may be conducted in this direction by which support systems can be developed for fostering diagnostic competence of pre-service mathematics teachers that helps to enhance the mathematics learning of struggling students.

Disclosure statement

No potential conflict of interest was reported by the author.

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