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Insights into Math Word Problems: Developing Learning Cycle for solving Word Problems

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Abstract

Word problems in mathematics are real-time problem situations, which can be solved using mathematical concepts. Inclusion of word problems in school mathematics curriculum aims to inculcate problem-solving skills among children. Word problems have always caught the attention of researchers because children face many learning challenges while working on word problems. This paper discusses the outcome of an empirical study conducted on fourth and fifth grade students to find out the learning difficulties they face and learning patterns they follow while working on word problems. George Polya's classical work (1945) on problem-solving and Anne Newman's seminal work (1977) on the working of word problems were used to develop conceptual framework for the study. An in-depth analysis was done to develop insights into the processes adopted by students at various levels of solving word problems. Findings of the study were used to construct suggestive learning framework for solving word problems.

Introduction

Word problem is defined as, "A mathematical problem that is stated in words rather than in symbols or as an equation" (*Mathematics Thesaurus*). Majority of mathematics topics contain word problems. The aim of word problems is to connect a mathematical concept with real life situation. Word problems act as linkages between concept and application. Mathematical word problems are focused to build the aptitude of problem-solving in children. School mathematics of the twenty-first

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century is viewed by educators as a subject, which should engage a learner in problem-solving and reasoning. It should also foster understanding and develop the learner's critical and analytical skills. Instruction should not be limited to plain mastery of algorithms or development of certain mathematical skills. It should involve learners in investigation through "exploring, conjecturing, examining and testing" (National Council of Teachers of Mathematics, 1990, p. 95). Successful problem-solving involves coordinating with previous experiences and knowledge to generate new representations and related patterns. In school, mathematics is the domain, which formally addresses problem-solving as a skill. Considering that it is an ability of use in one's life, techniques and approaches learnt in school have immense value. Mathematics also provides an opportunity to make interesting problems (National Curriculum Framework, 2005). It is experienced, particularly, in Indian schools that mathematical problem-solving is a major concern in a student's school life due to various reasons, such as lack of comprehension of the problem posed, strategic knowledge, domain-specific knowledge or experience in defining problems, inability to translate a problem into a mathematical form, tendency to rush towards a solution before the problem has been clearly defined. Therefore, word problems have always caught the attention of researchers to effectively blend problem-solving skills in children.

George Polya (13 December 1887–7 September 1985) is known as 'Father of Problem Solving'. He was a Hungarian mathematician. He worked as a professor of mathematics from 1914 to 1940 at ETH Zürich, and from 1940 to 1953 at the Stanford University. He made fundamental contributions to combinatory, number theory, numerical analysis and probability theory. He is also noted for his work in heuristics and mathematics education. George Polya's (1945) classical work has been published in his book *How to solve it*, Princeton University Press, 2004. It throws an insight into problem-solving and parameters involved in dealing with word problems. In this book, he identifies four basic parameters of problem-solving.

Table 1: Polya's four parameters for problem-solving

S.No.	*Parameter*	*Meaning*
1.	Understand the problem	Understanding all words used in stating the problem — what is being asked to find or show, restating the problem in own words, and illustrating the problem diagrammatically are enough to find a solution.
2.	Devise a plan	There are many ways to solve problems. Choosing an appropriate strategy to solve the problems is one such way.

3.	Carry out the plan	Persist with the plan that has been chosen. If it does not work, discard it and choose another.
4.	Look back	Take time to reflect on what has been done, what worked and what did not. Doing this will enable you to ascertain what strategy to use in order to solve future problems.

Inspired by the classical work of Polya, Australian educator Anne Newman (1977) suggested five significant parameters to help determine where errors may occur in students' attempts to solve written problems. She developed an 'Error Analysis Model' to classify errors and identified a sequence of steps. This model, known as 'Newman's Error Analysis (NEA) Model', has been used by many researchers to study word problems.

The parameters according to the Newman's Model are given in Table 2.

According to NCERT textbooks, word problems start at the upper primary grade (3rd grade onwards), which demand command over language, transformation, abstract concepts and fundamental knowledge to enable children to handle word problems effectively.

This empirical study dealt with the word problem-solving abilities of fourth and fifth graders. The study was explanatory in nature. An in-depth analysis was conducted to develop insights into the processes adopted by students at various levels of solving word problems. Findings of the study were used to construct suggestive learning framework for solving word problems.

Objectives of the Study

1. To find out which area of word problem students find the most difficult with reference to Newman's approach
2. To find the difference between the problem-solving approach

Table 2: Newman's five parameters of Error Analysis Model

S.No.	*Parameter*	*Meaning*
1.	Reading	Reading the problem
2.	Comprehension	Comprehending what is read
3.	Transformation	Carrying out a transformation from words of the problem to selection of an appropriate mathematical strategy
4.	Process	Applying the process skills as demanded by the strategy selected
5.	Encoding	Encoding the answer in an acceptable written form

of students of government and private schools with reference to Newman's approach

3. To find out the difference between the problem-solving approach of girls and boys with reference to Newman's approach
4. To construct suggestive learning framework for solving word problems

Methodology

It was a qualitative study, wherein children of fourth and fifth grades were surveyed to understand their thought processes while working with word problems and an in-depth analysis was conducted with reference to the Newman's Error Analysis Model using a 3–point rating scale.

Sample

Ten (five boys and five girls) government school students and 10 (five boys and five girls) private school students of Classes IV and V were selected as sample.

Five word problems were constructed after consulting the NCERT syllabus based on many criteria dealing with everyday life. The problems required fundamental knowledge, clubbing of one or more mathematical concepts, etc. Word problems were translated for Hindi-medium students. The conceptual framework of word problems is given in Table 3.

Due permission was taken by the school authorities and appropriate time was given to the students to solve the word problems. They were also provided with small clues as and when required.

Tool: A 3–point rating scale based on Newman's five steps was developed to assess the students' process of solving word problems.

Analysis

An in-depth analysis was conducted on the basis of Newman's Problem-solving Model to develop insights into the processes adopted by students at various levels of solving word problems. The analysis was mainly qualitative in nature but it was also quantitative in order to support the study.

Findings and Analysis of the Study

The students were marked on each Newman's parameter on the basis

Table 3: Conceptual framework of word problems

Word problem	*Underlying concepts*
Problem 1	Concept of calender, arithmetic operations
Problem 2	Concept of time or duration, arithmetic operations
Problem 3	Concept of money, arithmetic operations
Problem 4	Measurement of mass, money, arithmetic operations, estimation
Problem 5	Concept of temperature, arithmetic operations, estimation

of their effectiveness in dealing with word problems.

Findings

A 3–point rating scale was developed to judge all five parameters of Newman's Error Analysis Model, these are — reading the problems, comprehension of the problems, transformation of the problems, processing of the problems, and then, encoding the problems.

Table 4: 3–point rating scale used in the study

S.No.	*Effectiveness*	*Score*
1.	Was not able to do at all	0
2.	With errors	1
3.	Correctly	2

The maximum score for each parameter was 10 (two for each word problem). The minimum score for each parameter was 0 (0 for each word problem).

Table 5: Scoring of the sample

NEWMAN'S PARAMETERS Total score in all five word problems					
Students	*Reading*	*Comprehension*	*Transformation*	*Process*	*Encoding*
GM 1	9	7	6	3	0
GM 2	9	9	6	6	2
GM 3	9	9	8	5	1
GM 4	5	3	2	2	0
GM 5	8	8	5	1	0
GF 1	8	4	2	0	0
GF 2	7	6	2	1	0
GF 3	8	7	5	0	0
GF 4	9	8	5	3	2
GF 5	10	0	0	0	0
PM 1	9	7	5	6	6
PM 2	9	6	5	4	4
PM 3	10	6	5	4	5
PM 4	10	10	7	7	6
PM 5	10	10	10	8	8
PF 1	10	7	9	8	7
PF 2	10	8	9	8	7
PF 3	9	8	7	7	6
PF 4	10	10	10	9	9
PF 5	10	9	8	6	6

G: Government School *M: Male* *P: Private School* *F: Female*

Analysis

The data are compiled in the following histograms and a comparison can be drawn between the students.

1. Fig. 5.1 compares the score of boys and girls in a all five parameters in government school.
   Maximum score: 50
   Minimum score: 0
   The results indicate that in government school, boys outshined in all five Newman's processes of problem-solving.
2. The histogram given in Fig. 5.2 compares the score of boys and girls in all five parameters in a private school.
   Maximum score: 50
   The results indicate that in private schools, girls outshined boys in all five processes.
3. The histogram given in Fig. 5.3 compares the scores of government and private school students in all five parameters.

- The results indicate that children of private school performed better than those of government school when it came to solving word problems and going step-by-step through Newman's five parameters.
- The maximum gap between private and government school children was in the encoding process.
- Overall, the process of reading was achieved by most of the students (both girls and boys).
- But the maximum gap between girls and boys is in the processes of transformation and process skill.
- There is a significant lag in the process of comprehension in both the schools, which

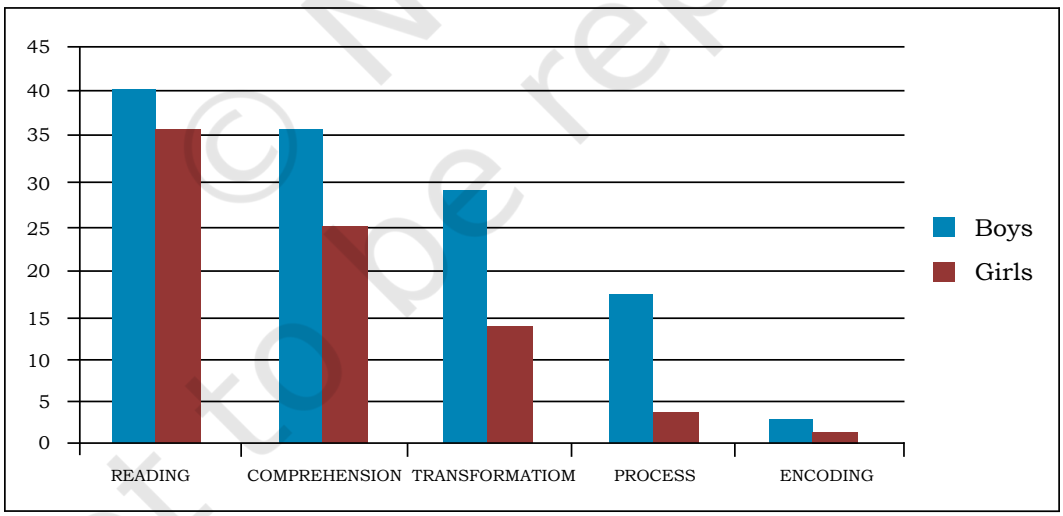

Fig. 5.1: Histogram comparing the scores of boys and girls of government school on Newman's parameters

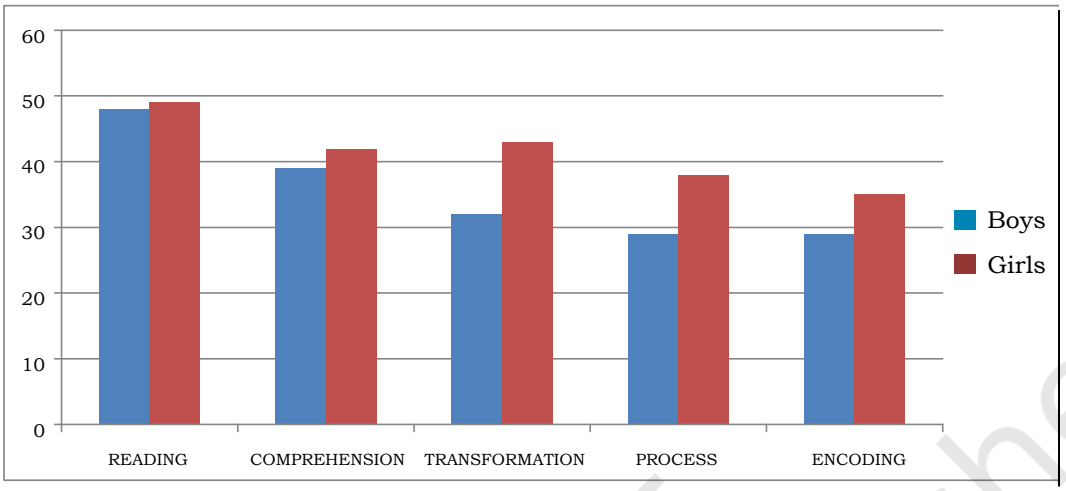

Fig. 5.2: Histogram comparing the scores of boys and girls of private school on Newman's parameters

Maximum score: 100

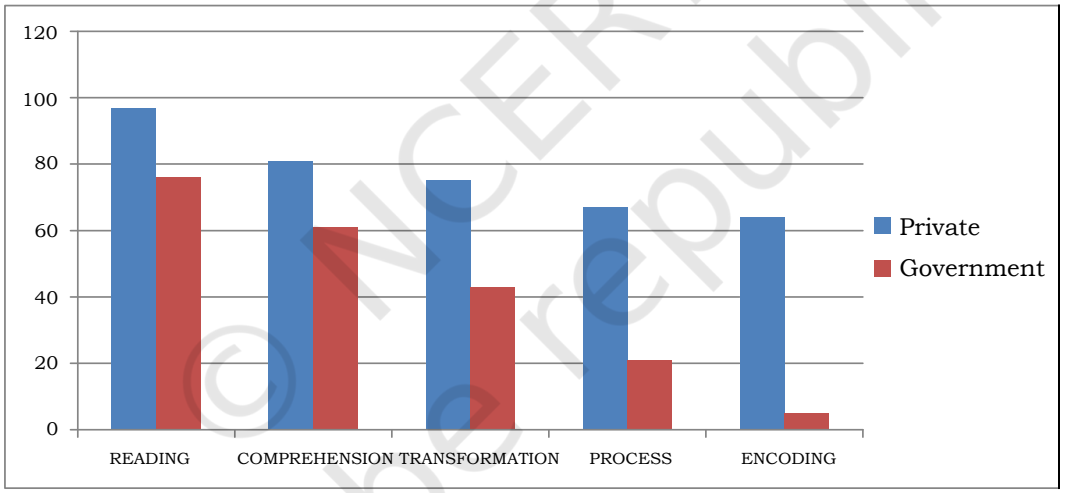

Fig. 5.3: Histogram comparing the scores of private and government school students on Newman's parameters

resulted in lags in other consecutive steps.

- The overall analysis among five parameters is

Reading process (most efficiency)> Comprehension > Transformation> Process Skill > Encoding (least efficiency)

CONCLUSION

Students have the habit of attempting to solve word problems using only one heuristic. They do not show flexibility in seeking to solve the problems using more than one heuristic. The

study highlights the processes, which students go through while working on word problems. Isolated questions without context create no interest in students and they are not able to solve the problems. It is seen that students are better in reading and comprehension processes but when it comes to converting or translating the words in mathematical terms to reach a solution, they are not able to do so. This is of great concern.

Students must possess the required knowledge and be able to use appropriate skills to solve problems. As the questions given to the students were in coordination with the National Curriculum Framework – 2005 and NCERT textbooks, there was still a lack of conceptual knowledge in some areas. Algorithmic knowledge, linguistic knowledge, conceptual knowledge, schematic knowledge and strategic knowledge are vital traits of developing problem-solving ability. For mathematics teachers to assist their students in developing problem-solving skills, it is essential that they are aware of their difficulties first. The study suggests a conceptual framework, which can help teachers to facilitate students on working with word problems in a better way. The framework consists of four domains:

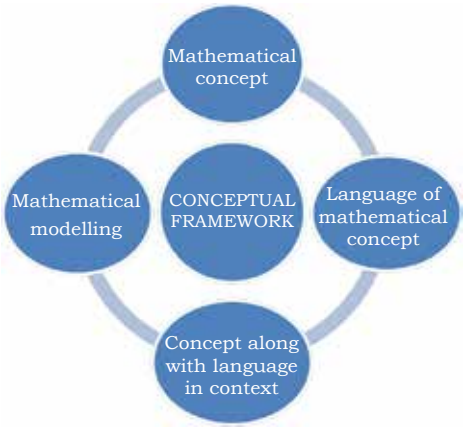

Fig. 5.4: Suggestive conceptual framework

- Mathematical concept includes introduction of concept, clarity in all concepts.
- Language of mathematical concept includes symbolic, verbal, pictorial, everyday context language.
- Concept along with language in context includes concept with real life.
- Mathematical modelling includes decoding, identifying the concepts and mathematical language, developing suitable algorithm, problem solving procedure, translating problem into solution.
- The framework is an attempt to help teachers and curriculum makers to build an insight on the child's perspectives of solving word problems.

REFERENCES

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