Effect of Modular Intervention on Eighth-Grade Students' Algebraic Reasoning
A Quasi-Experimental Study

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Abstract

The present study is a quasi-experimental investigation of the effect of a self-developed module on algebraic reasoning among eighth-grade students to improve their algebra learning. The participants were students from both public and private schools within the same district. To test the effectiveness of the module, four schools were selected—two assigned as experimental groups and two as control groups. Students in the experimental groups received modular teaching, while students in the control groups received traditional teaching. The findings were significant and provide valuable insights for classroom teachers and other stakeholders regarding the enhancement of algebraic learning. The results suggest that the module on algebraic reasoning was significantly effective in improving both algebraic reasoning and algebra achievement. It is recommended that the integration of substantial algebraic reasoning activities can have a positive impact on students' algebraic understanding and, more broadly, on mathematics learning at the middle-school level.

Keywords: Algebraic Reasoning, Quasi-Experimental, Algebra

Introduction

Mathematics learning remains a major concern across the globe. Across various grades and branches of mathematics, learners' achievement and understanding of fundamental concepts continue to be topics of debate. To address these persistent challenges in mathematics education, it is vital to clearly understand the nature of mathematics as a discipline, as this understanding supports

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effective teaching and higher learning (Dossey, 1992). The sequential nature of mathematics provides a foundation for the overall learning process, with each concept building upon the previous one.

Among all branches of mathematics, algebra presents a particularly high level of abstractness compared to the other areas. It is widely acknowledged as one of the most demanding domains of mathematics, especially at the upper primary and secondary school levels (Fuchs *et al.*, 2010; Wang, 2015; Stacy *et al.*, 2017). The essence of algebra lies in its structured use of symbols; most algebraic problems require step-by-step algorithmic solutions. Mastering algebra involves recognising patterns, understanding procedural knowledge, and grasping the sequential nature of problem-solving. Algebra opens new avenues for analytical thinking and enhances students' abilities to comprehend the reasoning behind diverse problem-solving strategies. However, its abstract nature often sets it apart from other branches of mathematics, posing significant challenges for students transitioning from concrete arithmetic operations to symbolic and generalised representations.

Despite various interventions and pedagogical approaches aimed at improving school algebra outcomes, research consistently shows that a large proportion of students struggle to learn algebra effectively (Wang, 2015). Many students encounter difficulties during the transition from arithmetic to algebra, particularly when dealing with symbolic representation, unknown numbers, variables, equations, functions, and other abstract concepts that demand higher-order thinking skills and reasoning (Konstantinos and Vosniadou, 2012).

At the upper primary level, the abstract nature of mathematics intensifies with the introduction of algebra. Algebra requires students to engage in problem-solving and reasoning, moving from the known to the unknown through symbolic manipulation. To meet these learning demands and enhance student achievement in algebra, fostering algebraic reasoning is key. Research indicates that algebraic reasoning and algebraic achievement are closely linked, with algebraic reasoning serving as a critical factor in students' algebraic development (Bazzini and Tsamir, 2004; Subramaniam and Banerjee, 2004; Lian and Yew, 2012; Jonsson *et al.*, 2014).

Furthermore, several studies highlight the effectiveness of modular teaching methods in developing various concepts at the school level (Singh, 2013; Missok, 2012; Charandas, 1990). Substantial research supports the use of a quasi-experimental pre-test post-test control group design to test the effectiveness of different teaching methods and strategies compared to conventional approaches in school subjects (Aydın *et al.*, 2018; Jhariya and Shinde, 2016; Rajesh, 2014; Jupri *et al.*, 2015).

Given these insights, the present study explores the impact of a self-developed modular intervention on algebraic reasoning among eighth-grade students, aiming to bridge the gap between arithmetic and algebra and enhance students' overall mathematical understanding.

Algebraic Reasoning: An Overview

Algebraic reasoning serves as a gateway for students progressing in mathematics and science (Greenes *et al.*, 2001; Wilkie, 2019, 2022). Developing algebraic reasoning—whether through problem-solving approaches, structured modules, or other classroom strategies—has proven to be an effective teaching practice for enhancing students' reasoning abilities and improving their achievement in algebra. The National Council of Teachers of Mathematics (NCTM, 2000) emphasises that middle school students must learn algebra not only as a set of concepts and skills related to representing quantitative relationships, but also as a way of thinking that enables them to formalise patterns and generalisations.

Algebraic reasoning is a process through which learners generalise mathematical ideas from specific cases, justify those generalisations through logical discourse, and express them in increasingly sophisticated, age-appropriate ways (Kaput and Blanton, 2005). It forms the essential foundation for developing abstract mathematical understanding and is a fundamental building block for cultivating mathematical ideas among learners. Algebraic reasoning is a powerful tool for mathematics teachers and other stakeholders concerned with improving mathematics education, especially at the stage when students are transitioning from concrete arithmetic to more abstract topics such as algebra and geometry.

At this transitional stage, students begin to encounter purely abstract concepts that demands higher-order thinking. Early development of algebraic reasoning builds students' capacity to reason skillfully with equations, functions, and variables. In mathematics learning, the process of reasoning is often more critical than the final product (Chaurasia, 2016). Algebraic reasoning plays a vital role in solving mathematical problems by promoting functional thinking, encouraging students to make conjectures, test ideas, and engage in reasoning and proving.

This process strongly emphasises the interconnectedness of key actions, including:

- Formulating conjectures to make and test proofs.
- Using the process of doing and undoing (reversibility) to verify solutions.
- Justifying and proving conjectures
- Predicting solutions in various ways and exploring multiple paths to arrive at answers

For example, many students who encounter an equation, such as 7 + 9 = ___ understand this as a prompt to compute and write 16 in the blank. However, when presented with an equation, like 7 + 9 = ___ + 6, they often still write 16 instead of the correct answer 10, revealing a limited understanding of the equal sign. Many learners treat the equal sign merely as a signal to calculate what is on the left, rather than as a symbol of balance between two sides of an equation. Likewise, when faced with an expression such as, $x + 5 = 0$, students may know how to handle the numerical operations but struggle with the presence of the variable x and the balancing nature of the equation.

This research highlights several common challenges students face when learning the algebra early, which can be grouped into five main categories:

1. **Difficulties in performing arithmetical operations within numerical and algebraic expressions:** Including problems adding or subtracting similar algebraic terms, using distributive, commutative, and inverse properties, and applying operations correctly (Herscovics and Linchevski, 1994; Linchevski, 1995; Booth, 1988; Bush and Karp, 2013; Warren *et al.*, 2013).
2. **Difficulties understanding variables and unknowns:** Students often struggle to grasp the concept of a variable as an unknown or changing quantity (Booth, 1988; Bush and Karp, 2013; Herscovics and Linchevski, 1994).
3. **Difficulties comprehending algebraic expressions:** Students may find it challenging to interpret and manipulate different algebraic forms (Arcavi, 1994; Tall and Thomas, 1991).
4. **Difficulties interpreting the equal sign:** Many students misunderstand its meaning and fail to apply it as a symbol of equivalence and balance (Bush and Karp, 2013; Herscovics and Linchevski, 1994; Kieran, 1981).
5. **Difficulties with mathematisation:** Learners often struggle to connect real-life situations with mathematical representations and vice versa (Treffers, 1987; Van den Heuvel-Panhuizen, 2003).

Addressing these challenges through well-designed interventions and targeted teaching methods is essential for strengthening students' algebraic reasoning and supporting their broader mathematical development.

Modular Approach of Teaching

Modules are designed primarily to promote active learning, focusing on fostering a deeper understanding of concepts among learners with the goal of facilitating effective learning. A module provides structured support to instructors through well-organised content aimed at bringing about the desired behavioral changes in learners. It achieves this by offering opportunities for learners to think independently and respond to various situations or activities in the module according to their own suitability. In essence, a module offers learner-centered opportunities that enables dynamic interactions with the provided content.

According to UNESCO (1988), a module is defined as a set of learning experiences planned around a specific topic, which includes elements such as instructional material, learning objectives, diverse teaching and learning activities, and assessment and evaluation conducted through criterion-referenced procedures.

The current paradigm in teaching and learning has shifted significantly, emphasising the need to make teaching more effective and learning more enjoyable. To achieve this, multiple instructional interventions and teaching methods — such as collaborative learning, project-based methods, laboratory work, and inquiry-based learning — are being widely adopted. Alongside these methods, there is a growing emphasis on making instruction systematic and learner-friendly. Modules have become an important tool in this regard.

The modular teaching method supports instructors by providing a clear roadmap to facilitate the learning process. It also grants learners greater freedom in knowledge construction and recognition. The modular approach is essentially an extension and improvement of the well-known concept of programmed instruction. This approach is grounded in the recognition of individual differences among learners and is specifically designed to cater to the unique needs of every learner involved in the educational process.

Key features of the modular teaching methods that distinguishes it from traditional lessons include:

- a pupil-centred design focus,
- content that may follow the textbook but is not strictly bound by it,
- a task-based approach to learning
- providing space for learner autonomy and exploration,
- promotion of integrated learning experiences, and
- systematic and ordered presentation of learning activities.

Concept and Development of Module on Algebraic Reasoning

The module on algebraic reasoning is a collection of units based on the algebra curriculum for Grade 8 students. It provides systematic learning experiences aimed at enhancing algebraic achievement and fostering algebraic reasoning among learners. The module is specifically focused on algebra and comprises a total nine units.

Table 1

Unit	Title of the unit
Unit 1	Fundamental Concepts of Algebra
Unit 2	Algebraic Expression
Unit 3	Algebraic Identity
Unit 4	Exponents and Powers
Unit 5	Factorisation I
Unit 6	Factorisation II
Unit 7	Linear Equations
Unit 8	Applications to Linear Equations
Unit 9	Introduction to Graphs

Development of the Units in the Module

The development of the units in the module was grounded in nine finalised dimensions of algebraic reasoning. These nine key dimensions are:

1. Doing–Undoing (Reversibility)
2. Abstracting from Computation
3. Building Rules to Represent Functions
4. Structural Understanding/ Understanding of Growing Patterns
5. Mathematical Generalisations
6. Understanding the Relationship between Operations and Functions
7. Analysing Patterns and Relationships
8. Use of Conjectures and Coordination of Numeric and Spatial Structures
9. Thinking about the Spatial Configuration of Sequence Terms or other structures

Elements of the Module on Algebraic Reasoning

Each unit in the module comprises the following elements:

1. Title
2. Introduction
3. Learning Objectives
4. Instructional Scheme
   - 4.1 Assigned Reading
   - 4.2 Assigned Writing
   - 4.3 Hands-on Activity
   - 4.4 Integration of Instructional Strategies Using the Dimensions of Algebraic Reasoning
5. Exercises/Activities
6. Assessment

The content for each unit was developed with primary focus on enhancing algebraic reasoning. The units were designed to foster algebraic reasoning alongside algebraic achievement by systematically incorporating the nine dimensions of algebraic reasoning into the content. The module's structure facilitates simultaneous development of both reasoning and achievement.

The investigator endeavored to develop each unit using well-organised activities, examples, elaborations, discussions, terminology explanations, and child-centered explanations of the content. Scaffolding of the content based on the dimensions of algebraic reasoning was frequently applied to promote a constructivist learning approach. All units were designed to be self-instructional to support independent learning.

Additionally, considering the experimental context and language needs of the students, the module was developed in two languages — English and Hindi. The English version was employed with students from CBSE schools, while the Hindi version was used for students from U.P. Board schools.

Validation of the Module

To validate the module, the investigator developed a "Proforma for Assessment of Module on Algebraic Reasoning," which included five key aspects for evaluation:

1. Content of the Module/Unit.
2. Organisation of the Content.
3. Language of the Content.
4. Presentation of the Content.
5. Overall Evaluation of the Module/Unit.

The finalised module was validated using this proforma. Each of the nine units was assigned to a different expert for evaluation. A total of 20 feedback responses were collected from experts in education, mathematics education, and language education.

The experts identified language ambiguities, offered pedagogical suggestions, and contributed ideas to improve and enrich the module. These suggestions were incorporated into the final draft. The majority of experts found the objectives, evaluation procedures, and content of the module to be appropriate and suitable for the learners' needs.

Example of Validation

In one of the units of the module titled Algebraic Identity, an algorithm for a detailed explanation was suggested to keep side by side in the content.

Consider an expression

$(x + 1)^2 = x^2 + 1 + 2x$

How?

Let us see $(x + 1)(x + 1)$

$= x \times x + x \times 1 + 1 \times x + 1 \times 1$

$= x^2 + x + x + 1$

$= x^2 + 2x + 1$

Reference: Unit 3-page no. XXI (Algebraic Identity)

Present Study

Various research studies and theoretical perspectives have established that algebraic reasoning is a vital component of mathematical learning, particularly for mastering algebra (Herscovics and Linchevski, 1994; Linchevski, 1995; Kieran, 1981; Treffers, 1987). Algebraic reasoning develops the learner's ability to generalise, enhances their capacity to handle symbols, and can be applied to a broad range of mathematical and real-life phenomena. It also helps to organise thinking about complex, interrelated ideas and serves as a tool for generalising mathematical concepts and identifying underlying structures. Based on these facts, the present study was conceptualised.

The study is designed to develop an instructional package for algebra in the form of a module that aims to foster algebraic reasoning among learners.

The fundamental assumption behind this effort is that, if algebraic reasoning can be nurtured at the early stages of elementary algebra, it will lay a strong foundation for more advanced mathematical learning in an organised and systematic manner (Carpenter and Levi, 2000).

Therefore, the purpose of developing an instructional package—the Module on Algebraic Reasoning (MAR)—is to provide teachers with a structured tool that makes algebra easier to teach and more effective for students to learn, thereby improving both their understanding and achievement in algebra. The underlying thought is that strengthening algebraic reasoning at the initial stages will create pathways for its continued development throughout higher mathematical learning.

Hence, the primary objective of the present study is to examine the effectiveness of the Module on Algebraic Reasoning (MAR) for Grade 8 students. The study also aims to investigate whether the use of this module can improve algebraic achievements and foster deeper algebraic reasoning. The findings of this study are expected to support the use of self-paced, student-centred teaching-learning modules that enhance algebraic reasoning and achievement among Grade 8 students.

Research Questions

The study is guided by two main research questions:

- Does the Module on Algebraic Reasoning foster algebraic reasoning among Grade 8 students in algebra?
- Is there a significant difference in the algebraic reasoning of students exposed to the modular teaching method compared to those taught using the traditional method?

Objectives of the Study

- To study the effectiveness of the Module on Algebraic Reasoning (MAR) in terms of algebraic reasoning among CBSE Board students of Grade 8.
- To study the effectiveness of the Module on Algebraic Reasoning (MAR) in terms of algebraic reasoning among UP Board students of Grade 8.

Hypotheses

H_{01}: There is no significant effect on the algebraic reasoning of CBSE Board students taught through the modular teaching method compared to those taught through the traditional method at the 0.05 level of significance.

H_{02}: There is no significant effect on the algebraic reasoning of UP Board students taught through the modular teaching method compared to those taught through the traditional method at the 0.05 level of significance.

Methodology of the Study

The main purpose of the study was to test the effectiveness of the developed module on algebraic reasoning for Grade 8 students. An experimental method was adopted to achieve this objective. The experiment was conducted in naturally occurring classroom settings. For this purpose, a non-equivalent pre-test–post-test quasi-experimental design was used, which is appropriate for real-world educational research where randomisation is not always feasible (Handley *et al.*, 2018; Liu *et al.*, 2020; Shadish *et al.*, 2001).

The intervention took place in actual mathematics classes in schools, with the researcher teaching all sections to maintain consistency. The classroom environments were conducive and appropriate for the experiment. Both the control and experimental groups were initially tested for equivalence using the following tools:

1. Intelligence Test by P.N. Mehrotra
2. Reasoning Test by L.N. Dubey
3. Test of Algebraic Reasoning developed by the researcher

These tests were administered during pre-testing to ensure that the experimental group (one CBSE and one UP Board school) and the control group (one CBSE and one UP Board school) did not differ significantly in terms of intelligence, general reasoning, or algebraic reasoning.

A pre-test was done to administer to both groups to establish baseline data and match the groups at initial level. After the pre-test, the experimental group received instruction using the modular teaching method with the developed module, while the control group was taught through the traditional teaching method. The duration of the experiment was 45 working periods, with each period lasting 35 minutes.

This quasi-experimental design allows for the non-random assignment of participants to the control and experimental groups—an approach commonly employed in educational research when randomisation is not feasible, but the study must be carried out in a natural setting.

The Experimental Design Used for the Study

The experimental design adopted for the study is presented as follows:

Group	Pre-test		Treatment	Post-test
A	O_1	⟶	X_1	O_2
B	O_1	⟶	X_2	O_2

A = Experimental, B = Control, X_1 = Modular Method, X_2 = Traditional Method, O_1 = Pre-Test, O_2 = Post-Test

Variables of the Study

The study involved two key variables:

(i) The independent variable was the teaching method, which included two modes: the modular teaching method and the traditional teaching method.

(ii) The dependent variable was the algebraic reasoning demonstrated by Grade 8 students.

Sample

For the study, the researchers selected a total of four schools: two UP Board schools (one assigned as the control group and one as the experimental group) and two CBSE schools (one assigned as the control group and one as the experimental group). To minimise the potential interaction effects during the intervention, the experimental and control groups were taken from different schools within the same locality, with similar learning environments, curricula, and standards. Previous research supports this approach, and evidence for its effectiveness is well documented in the literature (Aydın *et al.*, 2018; Jhariya and Shinde, 2016; Rajesh, 2014; Jupri *et al.*, 2015).

The final sample consisted of a total of 196 students, all enrolled in Grade 8. Due to institutional constraints, random assignment of students to groups was not permitted by the schools. Therefore, the study was conducted using intact classroom groups.

Table 1 presents the complete distribution of the sample across schools and groups:

Instrument

In the present study, to measure the independent and dependent variables, the investigators developed and standardised two main tools:

- Test of Algebraic Reasoning, and
- Module on Algebraic Reasoning (MAR).

These instruments were specifically designed to assess students' algebraic reasoning skills and to provide structured, systematic learning experiences to support the development of algebraic understanding.

Concept of the Module on Algebraic Reasoning

The Module on Algebraic Reasoning (MAR) is a structured set of learning units designed for Grade 8 students. It focuses on the core concepts of algebra and aims to provide systematic, step-by-step learning experiences that

Table 1

Type of schools	Experimental group	Total students in the experimental group	Control group	Total students in the control group	Total students
UP Board schools	Section B	65	Section A	51	116
CBSE Board schools	Section B	38	Section A	42	80

advance both algebraic reasoning and algebraic achievement. The module consists of a total of nine units, each carefully developed to cover key aspects of algebraic reasoning and to foster higher-order thinking among learners.

Table 2

Unit	Title of the unit
Unit 1	Fundamental Concepts of Algebra
Unit 2	Algebraic Expression
Unit 3	Algebraic Identity
Unit 4	Exponents and Powers
Unit 5	Factorisation I
Unit 6	Factorisation II
Unit 7	Linear Equations
Unit 8	Applications to Linear Equations
Unit 9	Introduction to Graphs

The Major Grounds for Module Development

The development of the module's units was firmly based on nine core dimensions of algebraic reasoning, selected to address critical areas where students often face challenges. These nine dimensions are:

1. Doing–Undoing (Reversibility).
2. Abstracting from Computation.
3. Building Rules to Represent Functions.
4. Structural Understanding/ Understanding of Growing Patterns.
5. Mathematical Generalisations.
6. Understanding the Relationship between Operations and Functions.
7. Analysing Patterns and Relationships.
8. Use of Conjectures, Coordination of Numeric and Spatial Structures.
9. Thinking about the Spatial Configuration of Sequence Terms or Other Structures.

Aims of the Module

The module was designed with clearly defined aims, aligned with the needs of learners and the nature of the content. These aims served as a guide for setting learning objectives and planning a series of flexible activities and experiences to achieve the desired outcomes. The specific aims of the module are:

- To develop students' interest in learning algebra through engaging text readings and activities.
- To improve students' overall learning and understanding of algebraic concepts.
- To foster algebraic reasoning among students.
- To enhance algebra learning while simultaneously strengthening students' ability to reason algebraically.

A total of nine units were developed to meet these aims. Each unit includes objectives and instructions explicitly designed to promote algebraic reasoning. To address the linguistic diversity of learners, the module was also translated into Hindi for use with UP Board students.

About the Test of Algebraic Reasoning (TAR)

To evaluate the effectiveness of the developed module, a criterion-referenced test of algebraic reasoning was developed by the investigator. This test measures students' algebraic reasoning skills, drawing its items directly from the Grade 8 algebra curriculum. Each item in the test was constructed to align with and assess one or more of the nine dimensions of algebraic reasoning identified in the module.

Reliability of the Test of Algebraic Reasoning

The reliability of the Test of Algebraic Reasoning (TAR) was established using two methods—the split-half reliability method and the Kuder-Richardson Formula 20 (K-R 20). The test was also reviewed for face validity and content validity. The internal consistency reliability was found to be strong, with Cronbach's α values ranging from 0.74 to 0.81 (Wu, 2002). The detailed results of the reliability analysis are presented in the table below.

Validity of the Test of Algebraic Reasoning

To establish the validity of the *Test of Algebraic Reasoning*, content validity was determined.

Content Validity

The content validity of the test was assessed based on the expert judgment and feedback from professionals in the fields of education, mathematics, and mathematics education. All test items were thoroughly examined by these experts to ensure relevance, alignment with learning objectives, and appropriateness for Grade 8 students. Additionally, the relevancy of the items was verified during the first tryout phase.

Based on expert feedback and preliminary testing, the Test of Algebraic Reasoning can be considered to have satisfactory content validity.

Experiment

The experiment was conducted in a real classroom setting, using the regular mathematics periods in the selected schools. The Test of Algebraic Reasoning was administered on both the control and experimental groups for pre-testing. The experimental groups (one CBSE and one UP Board

Table 3

Measuring tool	Reliability coefficient	
Test of Algebraic Reasoning	Split-half Reliability of Test of Algebraic Reasoning	K-R 20 Reliability of Test of Algebraic Reasoning
Value of reliability coefficient	0.88	0.87

school) were taught using the modular teaching method, while the control groups (one CBSE and one UP Board school) received instructions through traditional methods.

This quasi-experimental design is well-suited for educational research as it accounts for practical limitations such as non-random assignment and natural classroom settings, while still allowing the researcher to assess the effects of the intervention accurately (Cousins *et al.*, 2014; Epel *et al.*, 2019; Erica *et al.*, 2022).

The intervention was completed over 45 working periods, each lasting 35 minutes for both groups. The experimental group received instruction through the modular teaching method, covering all nine units within the given timeframe, while the control group was taught the same content using traditional teaching practices. After the instructional period, a post-test was administered to both groups using the same *Test of Algebraic Reasoning*.

Traditional Instruction for the Control Group

The control groups were taught the same content covered in the module, but through conventional teaching methods. Each concept was taught sequentially in a typical classroom setting without the structured, student-centered, and activity-based approach of the modular method.

Data Collection and Analysis

To evaluate the effectiveness of the developed module, data were collected from four schools: two CBSE board schools and two UP board schools. To arrive at meaningful conclusions, various statistical methods were employed, including Mean, Standard Deviation (SD), Levene's Test for equality of error variances, and Analysis of Covariance (ANCOVA).

Levene's Test of Homogeneity was used to check for equal variances between the groups (Levene, 1960). ANCOVA was then applied to examine differences in mean post-test scores while controlling for pre-test scores. Using ANCOVA instead of repeated measures ANOVA helped to minimise hidden variability and increase the power of the test, producing more precise results (Brown and Forsythe, 1974; Owen and Froman, 1988).

Results

To investigate the first objective—studying the effectiveness of the Module on Algebraic Reasoning (MAR) in terms of the algebraic reasoning of CBSE board students of Grade 8—an Analysis of Covariance (ANCOVA) was conducted.

Table 4
Descriptive Statistics of CBSE Board Schools for the Test of Algebraic Reasoning

(Dependent variables: Post-test)

School	Group	N	Mean	S.D.
School 1	Control	38	14.16	5.65
School 2	Experimental	42	16.74	5.09

Interpretation

Table 4 shows that there were 38 students in the control group, who were taught using the traditional method of teaching, and 42 students in the experimental group, who were taught using the modular teaching method with the help of the developed module.

It is evident from Table 4 that the mean post-test score for algebraic reasoning in the group taught through the traditional method was 14.16 with a standard deviation of 5.65, while the mean post-test score for the group taught through the modular teaching method was 16.74, with a standard deviation of 5.09.

These descriptive results suggest that students who received modular instruction achieved higher mean scores in algebraic reasoning compared to those taught through traditional methods.

Testing the Assumption of Independence Between Treatment and Covariate (Pre-Test)

Before applying Analysis of Covariance (ANCOVA), it is necessary to verify the fundamental assumption that there is no significant interaction between the treatment (teaching method) and the covariate (pre-test scores). This ensures that the relationship between the pre-test and post-test scores is consistent across groups, and the covariate does not interact with the treatment condition.

To test this assumption, an ANOVA was employed to check for any significant interaction between the pre-test scores and the treatment conditions. Additionally, independent samples t-tests can also be used for cross-verification of this assumption. The results of these tests confirmed that there was no significant interaction between the treatment and the covariate, validating the use of ANCOVA for further analysis.

Table 5
Independence of Treatment and the Covariate (pre-test) for the Data of CBSE Board Schools for the Test of Algebraic Reasoning

Test	Group	N	Mean	S.D.	F-value	p-value	Decision
Test of Algebraic Achievement	Control	38	11.00	5.72	0.148	0.702	Not significant at a 0.05 level of significance
	Experimental	42	11.48	5.36			

Interpretation

Table 5 clearly shows that the F-value for the interaction between the treatment and the covariate (pre-test) is 0.148, with p = 0.702, which is greater than 0.05. This indicates that the interaction is not statistically significant at the 0.05 level of significance. Therefore, it can be concluded that the treatment effect and the covariate (pre-test scores) are independent of each other. Hence, this assumption for employing ANCOVA on the CBSE schools data is satisfied.

Testing the Assumption of Homogeneity of Regression Slopes

Another key assumption for ANCOVA is the homogeneity of regression slopes. This assumption ensures that the relationship between the covariate (pre-test scores) and the dependent variable (post-test scores) is consistent across treatment groups, indicating that the effect of the covariate is uniform.

The results of the homogeneity of regression slopes test were not significant (p = 0.450), indicating that there is no significant interaction between the covariate and the treatment groups. Therefore, this assumption is also met, and it is appropriate to use ANCOVA for the analysis.

Testing the Assumption of Homogeneity of Variance

The homogeneity of variance is another fundamental assumption underlying the validity of parametric tests such as the t-test, ANOVA, and ANCOVA. This assumption means that the dependent variable should have equal variance across groups—in other words, the error variance should be similar for both the control and experimental groups.

Levene's Test is commonly used to test this assumption. If Levene's test is found to be non-significant, it indicates that the error variances are equal across groups, confirming that the assumption holds true. In this study, Levene's test was employed to test the homogeneity of variance for the post-test scores between the control and experimental groups.

Table 6
Levene's Test of Equality of Error Variance

Post-test	F	df 1	df 2	p-value	Decision
Test of Algebraic Reasoning	0.546	1	78	0.462	Not significant at 0.05 level of significance

For this study, the homogeneity of variance assumption between the two groups was tested using Levene's test for the post-test scores on the Test of Algebraic Reasoning (TAR). The result yielded p = 0.462, which is greater than 0.05. This indicates that the assumption of homogeneity of variance is satisfied.

Therefore, since all three assumptions—independence of treatment and covariate, homogeneity of regression slopes, and homogeneity of variance—are met, it can be concluded that the data fulfil the conditions required to apply ANCOVA.

Consequently, ANCOVA was used to test the effectiveness of the modular teaching method on the algebraic reasoning of Grade 8 students in CBSE board schools.

Table 7

Effectiveness of the Module on Algebraic Reasoning (MAR) in Terms of Algebraic Reasoning of UP Board Students of Grade 8

GROUP	Pre-test mean	Post-test mean	Source of variation	df	Sum of squares	Mean sum of squares	Fy.x	p-value
Control	11.00	14.16	Among	1	93.287	93.287	16.995	.000<.05
Experimental	11.47	16.74	Within	77	422.673	5.489		

Table 8
Descriptive Statistics of UP Board Schools for the Test of Algebraic Reasoning

(Dependent variables: Post-test)

School	Group	N	Mean	S.D.
School 1	Control	51	8.29	2.69
School 2	Experimental	65	12..26	3.50

Interpretation

Table 7 shows that the Analysis of Covariance (ANCOVA) for the CBSE Board students was significant, $F(1, 77) = 16.995$, $p = 0.000$, which is less than 0.05. This indicates that the difference in the adjusted post-test means between the control and experimental groups is statistically significant at the 0.05 level of significance ($df = 1, 77$). Therefore, the null hypothesis is rejected.

Furthermore, the mean post-test score on the Test of Algebraic Reasoning (TAR) for the experimental group ($M = 16.74$) was significantly higher than that of the control group ($M = 14.16$). This demonstrates that students who were taught using the modular teaching method performed significantly better in algebraic reasoning compared to those who received traditional instruction.

Thus, it may be concluded that the Module on Algebraic Reasoning (MAR) had a positive and significant impact on improving the algebraic reasoning of CBSE Board Grade 8 students.

Interpretation

It is evident from Table 8 that the mean score of the post-test for the group taught using the traditional method was M = 8.29 with a standard deviation of 2.69, whereas the group taught using the modular teaching method achieved a mean score of M = 12.26 with a standard deviation of 3.50. This descriptive comparison suggests that the students exposed to the Module on Algebraic Reasoning (MAR) outperformed those taught using the traditional method.

Testing the Assumption of Independence of Treatment and the Covariate (Pre-test)

Before conducting ANCOVA, it is essential to ensure that the assumption of independence between the treatment and the covariate (pre-test scores) is satisfied. This means there should be no significant interaction effect between the treatment condition and the covariate. To verify this, an ANOVA was employed to test the interaction.

Table 9

Independence of Treatment and the Covariate (pre-test) for the Data of UP Board Schools for the Test of Algebraic Reasoning

Test	Group	N	Mean	S.D.	F value	p-value	Decision
Test of Algebraic Reasoning	Control	51	7.75	2.87	1.190	0.278	Not significant at 0.05 level of significance
	Experimental	65	8.35	3.07			

Interpretation

Table 9 shows that the F value for the interaction between the treatment and the covariate (pre-test) is 1.190, with p = .278, which is greater than 0.05. This non-significant result indicates that there is no interaction effect between the teaching method and the pre-test scores. Therefore, the assumption of independence between the treatment and the covariate for employing ANCOVA on the UP Board schools' data is satisfied.

Testing the Assumption of Homogeneity of Regression Slopes

The homogeneity of regression slopes assumption ensures that the relationship between the covariate (pre-test scores) and the dependent variable (post-test scores) is consistent across both groups. The test for homogeneity of slopes returned a non-significant result ($p = 0.485$), confirming that this assumption is also met.

Testing the Assumption of Homogeneity of Variance

The assumption of homogeneity of variance requires that the error variances of the dependent variable are equal across groups. Levene's test was used to assess this assumption. A non-significant outcome indicates that the variances are equal, thereby satisfying this assumption and justifying the use of ANCOVA.

Since all three key ANCOVA assumptions—independence of treatment and covariate, homogeneity of regression slopes, and homogeneity of variance—are met for the UP Board data, ANCOVA can be validly applied to evaluate the effectiveness of the modular teaching method on students' algebraic reasoning.

Table 10
Levene's Test of Equality of Error Variance

Post-test	F	df 1	df 2	p-value	Decision
Test of Algebraic Reasoning	6.258	1	114	0.014	Not significant at 0.05 level of significance

Table 11

GROUP	Pre-test mean	Post-test mean	Source of variation	df	Sum of squares	Mean sum of squares	Fy.x	p-value
Control	7.75	8.29	Among	1	385.540	385.540	46.463	.000<.05
Experimental	8.35	12.26	within	113	937.656	8.298		

For this dataset, the assumption of homogeneity of variance between the two groups is met for the Test of Algebraic Reasoning post-test (p = 0.014 > 0.05). Hence, the homogeneity of variance assumption is satisfied. Therefore, all assumptions for conducting ANCOVA are fulfilled, and it can be concluded that ANCOVA can be appropriately employed for analysing the data of UP Board schools on the Test of Algebraic Reasoning (TAR).

Consequently, ANCOVA was used to test the effectiveness of the modular teaching method on the algebraic reasoning of Grade 8 students of the UP Board.

ANCOVA showing pre-test and post-test scores of the control and experimental groups of UP Board schools on the Test of Algebraic Reasoning (TAR).

Interpretation

Table 11 shows that the Analysis of Covariance (ANCOVA) was significant, $F(1, 113) = 46.463$, $p < .005$ (see Table 11). The obtained value *Fy.x* (46.463) for the difference in the post-test scores was found to be significant at the 0.05 level of significance with *df* = (1, 113). Hence, the null hypothesis was rejected.

The mean post-test score on the Test of Algebraic Reasoning (TAR) for the experimental group (M = 12.26) was significantly higher than that of the control group (M = 8.29) for the UP Board school. This demonstrates that the post-test mean of the experimental group is greater than that of the control group. Therefore, it is evident that the Module on Algebraic Reasoning (MAR) has significantly contributed to enhancing the algebraic reasoning abilities of UP Board students.

Experiences of Students Regarding the Module on Algebraic Reasoning (Qualitative Inquiry)

To further explore students' perspectives, the researcher developed an *Opinionnaire on the Module on Algebraic Reasoning* to capture the experiences of students who were taught using the developed module. After the completion of the post-test in both groups across all schools, this opinionnaire was administered exclusively to the experimental groups of both the CBSE and UP Board schools.

The opinionnaire contained 10 open-ended questions focusing on key aspects of the module and aimed to collect students' reflections on their learning experiences. The responses from students in both experimental groups were categorised into four major themes:

(i) Experiences related to overall learning through the module

(ii) Experiences related to the activities included in the module

(iii) Experiences related to learning mathematics, especially algebra, through the module

(iv) Experiences comparing learning through the module with learning through a traditional textbook

Overall, the students' feedback was overwhelmingly positive and encouraging. Many students described the module as engaging and resourceful in terms of content, delivery, activities, and support for mathematics learning. They noted that the module made learning algebra more interesting and accessible, particularly for challenging topics such as polynomials, linear algebra, factorisation, word problems based on linear equations, and concepts related to graphical solutions.

Students appreciated that the concepts were explained clearly within the various units of the module and found the learning objectives useful for understanding what they were expected to learn in each unit. They also highlighted the value of the activities embedded in the module, noting that these helped them practice and reinforce their algebraic reasoning skills in an interactive way.

Several students mentioned that, compared to learning through a traditional textbook, the module provided clearer explanations, better structure, and more opportunities for hands-on practice and reflection. Some students even suggested that similar modules should be developed for other areas of mathematics and other subjects.

In summary, students reported enjoying learning mathematics through the module and acknowledged its effectiveness and practical utility. Their positive experiences align with the quantitative findings, as reflected in the significant mean gains in the Test of Algebraic Reasoning (TAR). The students' feedback affirms that the module was successful in fostering both engagement and understanding in algebra learning.

Some Responses of the Students

Opinionnaire on Module of Algebraic Reasoning

Name.... Palak Singh .... **Class** ..... 8th A ....

School.... N.K.P.S. ....

Q. 1 How do you like every units of module?

प्रश्न1. मॉड्यूल की हर इकाई आपको कैसे पसंद आया?

मॉड्यूल की हर इकाई हमें बहौत पसंद आयी क्योंकि हर मॉड्यूल अलग - अलग बातें दी गई थी । और हर चीज़ बुक से अलग थी । इस मॉड्यूल से हम बहुत कुछ सीखे ।

Q.2 Which activity in the each unit you find very new to you?

प्रश्न 2. प्रत्येक इकाई में आपको कौन सी गतिविधि ऐसी मिली जो आपको बहुत नई लगी है?

Application to linear Equation and all units are best.

Q.3 Way of explaining the contents in the each unit was similar to your regular textbook or different from your regular textbook if yes then in what ways?

प्रश्न 3. प्रत्येक इकाई में प्रत्ययों को समझाने का तरीका आपकी नियमित पाठ्यपुस्तक के समान था या आपकी नियमित पाठ्यपुस्तक से अलग था यदि हां तो किस तरह से?

हाँ, ये पाठ्यपुस्तक से बहोत अलग था इसमें ऐसे - ऐसे तरीके दिये गए थे जो हमारी किताबों मे नही थी । इसलिए मुझे मे पढ़ने मे मज़ा आया ।

Q.4 Which activity of units do you like the most and why?

प्रश्न 4. इकाइयों की कौन सी गतिविधि आपको सबसे ज्यादा पसंद आयी है और क्यों?

unit - 8

Q.5 Does each unit of modules provide you the full explanation of contents?

प्रश्न 5. क्या मॉड्यूल की प्रत्येक इकाई आपको प्रत्ययों का पूर्ण स्पष्टीकरण प्रदान करती है?

1

Opinionnaire on Module on Algebraic Reasoning

आयत, वर्ग, त्रिभुज भी बहुत अच्छा लगता था पढ़ने में।

Q.5 Does each unit of modules provide you the full explanation of contents?

प्रश्न 5. क्या मॉड्यूल की प्रत्येक इकाई आपको प्रत्ययों का पूर्ण स्पष्टीकरण प्रदान करती है?

प्रत्येक ईकाई में प्रश्नों को पहचानने हल करने तथा उन्हें जानने में स्पष्टीकरण करने में इत्यादि चीजों को सिखने में अच्छा लगा।

Q.6 Does each unit contain some special types of tricks and hints for elaborating the concepts?

प्रश्न 6. प्रत्ययों को विस्तारित करने के लिए प्रत्येक इकाइयों में कुछ विशेष प्रकार की चाल और संकेत होते थे? YES/NO

Q. 7 Which unit was related to your daily life?

प्रश्न 7. कौन सी इकाई आपके दैनिक जीवन से संबंधित थी?

हमारे दैनिक जीवन में जो काम आने वाले चीजें थे उसमें कुछ ईकाइयों में प्रश्न कुछ प्रश्न होते थे।

Q.8 Would you like to study other subject with help of similar module?

प्रश्न 8. क्या आप इसी तरह के मॉड्यूल की मदद से अन्य विषय का अध्ययन करना चाहेंगे?

हम इस तरह के मॉडल की तरह, विज्ञान, अंग्रेजी का भी अध्ययन करना चाहते हैं कि हमारे लिए कुछ नया सिखने को मिला।

Q.9 Which components of the module you like the most?

प्रश्न 9. मॉड्यूल के कौन से घटक आपको सबसे ज्यादा पसंद करते हैं?

मॉडल में मुझे समीकरण और सर्वसमिकाएँ सबसे ज्यादा पसन्द आया फिर बीजिय व्यजक।

Q.10 Do you enjoyed learning mathematics, especially algebra with the help of module?

प्रश्न 10. क्या आपने मॉड्यूल की मदद से गणित, विशेष रूप से बीजगणित सीखने का आनंद लिया?

हाँ क्योंकि हमारे विद्यालय में बीजगणित सीखने के लिए इस तरह का आनन्द नहीं आता था।

2

Opinionnaire on Module on Algebraic Reasoning

Name..Namrata..Upadhyay.................. **Class** ...8th..A.........
School..Navneeta...Kunwar Public School..............................

Q. 1 How do you like every units of module?
प्रश्न1. मॉड्यूल की हर इकाई आपको कैसे पसंद आया?

....I..like..every..units..of..the..module..because..we..have..learnt
...many..things..that..is..not..given..in..book................

Q.2 Which activity in the each unit you find very new to you?
प्रश्न 2. प्रत्येक इकाई में आपको कौन सी गतिविधि ऐसी मिली जो आपको बहुत नई लगी है?

....I..find..Unit..1..and..unit..8..very..new..to..me:................

Q.3 Way of explaining the contents in the each unit was similar to your regular textbook or different from your regular textbook if yes then in what ways?
प्रश्न 3. प्रत्येक इकाई में प्रत्ययों को समझाने का तरीका आपकी नियमित पाठ्यपुस्तक के समान था या आपकी नियमित पाठ्यपुस्तक से अलग था यदि हां तो किस तरह से?

...Yes..~~if~~..it..wast..different..from..our..regular..textbook
......because..there..were..also..some..other..important..thing..that
.....was..not..given..in..the..textbook:......................

Q.4 Which activity of units do you like the most and why?
प्रश्न 4. इकाइयों की कौन सी गतिविधि आपको सबसे ज्यादा पसंद आयी है और क्यों?

.....I..like..unit-7..the..linear..equation..very..much..because
..it..was..different..from..text..book:..........................

Q.5 Does each unit of modules provide you the full explanation of contents?
प्रश्न 5. क्या मॉड्यूल की प्रत्येक इकाई आपको प्रत्ययों का पूर्ण स्पष्टीकरण प्रदान करती है?

~~Yes~~

1

Opinionnaire on Module on Algebraic Reasoning

का नाम पता चला उनका नाम रैने देकार्ते था।

Q.5 Does each unit of modules provide you the full explanation of contents?
प्रश्न 5. क्या मॉड्यूल की प्रत्येक इकाई आपको प्रत्ययों का पूर्ण स्पष्टीकरण प्रदान करती है?

प्रत्येक इकाई में प्रश्नों को पहचानने में कर हल कर सकते थे। उसमें उदाहरण → दिया था।

Q.6 Does each unit contain some special types of tricks and hints for elaborating the concepts?
प्रश्न 6. प्रत्ययों को विस्तारित करने के लिए प्रत्येक इकाइयों में कुछ विशेष प्रकार की चाल और संकेत होते थे ? YES/NO ✓

Q. 7 Which unit was related to your daily life?
प्रश्न 7. कौन सी इकाई आपके दैनिक जीवन से संबंधित थी?

प्रत्येक ~~इकाइ~~ इकाई में सवाल अच्छे थे। उसमें से सब हमारे दैनिक ~~जी~~ जीवन से संबंधित थे। दैनिक जीवन में काम आने वाली इकाई थी।

Q.8 Would you like to study other subject with help of similar module?
प्रश्न 8. क्या आप इसी तरह के मॉड्यूल की मदद से अन्य विषय का अध्ययन करना चाहेंगे?

हाँ हम इसी तरह के मॉड्यूल की मदद से अन्य विषय का अध्ययन करना चाहेंगे।

Q.9 Which components of the module you like the most?
प्रश्न 9. मॉड्यूल के कौन से घटक आपको सबसे ज्यादा पसंद करते हैं?

मॉड्यूल के सर्वसमिका चर अचर पर सबसे ज्यादा पसंद आयी।

Q.10 Do you enjoyed learning mathematics, especially algebra with the help of module?
प्रश्न 10. क्या आपने मॉड्यूल की मदद से गणित, विशेष रूप से बीजगणित सीखने का आनंद लिया?

हाँ हम मॉड्यूल की मदद से गणित, विशेष रूप से बीजगणित सीखने का आनंद लिया।

2

Discussion

The present study aimed to develop a module on algebraic reasoning and test its effectiveness among Grade 8 students. The study addressed two main objectives, examined through two null hypotheses. These hypotheses were tested using rigorous statistical methods, and the results clearly indicate that students in the experimental groups showed significantly higher gains than those in the control groups in both CBSE and UP Board schools. The mean scores reveal that students taught using the modular teaching method performed better in algebraic reasoning than those taught by the traditional method.

It can therefore be concluded that the developed module on algebraic reasoning was effective in enhancing the algebraic reasoning abilities of Grade 8 students. The findings align with similar studies that have demonstrated the benefits of modular and activity-based teaching approaches (Singh, S., 2013; Missok, J., 2012; Charandas, B., 1990). Moreover, the results reinforce the well-established relationship between algebraic reasoning and algebraic achievement (Bazzini and Tsamir, 2004; Subramaniam and Banerjee, 2004).

Overall, the study supports the argument that the modular teaching method can be an effective alternative of conventional teaching methods for fostering deeper understanding and reasoning in algebra at the middle school level.

Limitations of the Study

The conclusions drawn from this study should be interpreted with the following limitations in mind:

(i) The study employed a quasi-experimental design, which involved intact groups rather than random assignment. This may mean that the groups differed in ways not controlled for by the researcher. However, the use of ANCOVA minimised the potential impact of such pre-existing differences.

(ii) The findings may not be generalisable to all student populations, as the study was conducted in specific schools with particular demographic contexts. Future research should verify these results in varied educational settings.

Recommendations for Future Research

Based on the findings and experience gained from this study, the following recommendations are proposed for future research:

(i) Replicate the study with different education boards and grade levels.

(ii) Compare the modular approach with other innovative teaching methods.

(iii) Develop and test modules in multiple languages to address diverse linguistic contexts.

(iv) Extend this framework to other areas of mathematics to foster reasoning and achievement.

(v) Conduct studies with random assignment of participants to strengthen experimental rigor.

(vi) Explore the generalisability of the findings through larger and more diverse samples.

Implications

The conclusions of this study hold valuable implications for students, teachers, teacher educators, and curriculum developers:

(i) The developed module can be used as a self-learning resource to supplement regular classroom teaching.

(ii) Teachers may adopt the modular approach as an effective alternative to traditional methods for improving both algebraic reasoning and achievement.

(iii) The module, currently available in Hindi and English, could be translated into other regional languages to expand its reach.

(iv) The module may serve as a remedial tool for students who need additional support in algebra.

(v) Teacher education programmes can integrate training in modular teaching to equip future teachers with effective strategies for fostering reasoning skills.

(vi) Curriculum designers may consider adopting a modular format for textbooks to encourage self-paced and active learning.

Conclusion

Based on the statistical analyses and findings of this study, the following conclusions are drawn:

(i) **Conclusion 1:** The module on algebraic reasoning (MAR) was significantly more effective than the traditional method in developing algebraic reasoning among Grade 8 CBSE board students.

(ii) **Conclusion 2:** The module on algebraic reasoning (MAR) was significantly more effective than the traditional method in developing algebraic reasoning among Grade 8 UP Board students.

In summary, the developed module proved effective in enhancing algebraic reasoning among Grade 8 students from both CBSE and UP Boards. The findings highlight that targeted classroom interventions using well-designed modules can strengthen students' foundational understanding in algebra, consistent with previous studies on modular approaches in mathematics education (Singh, S., 2013; Missok, J., 2012; Charandas, B., 1990; Aydın *et al.*, 2018; Jhariya and Shinde, 2016; Jupri *et al.*, 2015; Bazzini and Tsamir, 2004; Lian and Yew, 2012; Jonsson *et al.*, 2014).

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