Concept Attainment in Mathematics and Its Predictors

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Abstract

Mathematics is an important subject of school curriculum because it is a science which draws necessary conclusions for cultivating our civilizations and developing intellectual values in human being. Mathematics is full of concepts. Basic to mathematics learning is the attainment of mathematical concepts. Without attaining concepts in mathematics, mathematical success cannot be achieved. The present study was designed to find the contribution of logical reasoning, mathematical creativity, socio-economic status and concept attainment model on attainment of mathematical concepts. The findings of present study show the predictive efficiency and contribution of predictors on attainment of concepts in mathematics.

सार

गणित, स्कूल पाठ्यक्रम का एक महत्वपूर्ण विषय है क्योंकि यह एक ऐसा विज्ञान है जो हमारी सभ्यता और मानव में बौद्धिक मूल्यों को विकसित करने के लिए आवश्यक ज्ञान प्रदान करता है। गणित अवधारणाओं से भरा है। गणित सीखने के लिए गणितीय अवधारणाओं की प्राप्ति आवश्यक है। गणित में अवधारणाओं को प्राप्त किए बिना, गणितीय सफलता हासिल नहीं की जा सकती। वर्तमान शोध में गणितीय अवधारणाओं की प्राप्ति पर तार्किक चिंतन, गणितीय सर्जनात्मकता, सामाजिक-आर्थिक स्थिति और अवधारणा प्राप्ति मॉडल के योगदान का अध्ययन किया गया है। वर्तमान अध्ययन के निष्कर्ष गणित में अवधारणाओं की प्राप्ति पर उपर्युक्त कारको के प्रभाव को दर्शाते हैं।

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Introduction

Mathematics is the numeric and calculative part of the human life. It enables an individual to study various phenomena in space and establishes relationships among them. The learning of mathematics at primary and secondary level is, therefore, necessary for the development of human resources of any nation and is the foundation of higher education and research. But, many researchers have reported that mathematics education has remained in dilapidated condition all over the country. Rastogi (1983) observed backwardness in basic arithmetic skills. Das (1998) studied the assessment of class IX students on basic mathematics and found that the level of achievement was up to 47 per cent only. *National Achievement Survey* (2017) reported that learning outcomes in mathematics of class-VIII students is not very satisfactory across the country. Annual Status of Education Report- ASER (2018) released by 'PRATHAM' also revealed the same scenario.

Mathematics, as an educational discipline, is full of concepts. Most of the concepts are abstract in nature and are represented by the abstract signs and symbols. Mathematics helps individual to think logically. Basically, mathematics is a logically organized conceptual system (Ernest, 1991). But, the teaching of mathematics is not basically connected with conceptual understanding. Students learn mathematics by rote learning and solve mathematical problems with the help of calculation ability and by practising. Studies revealed that achievement level of students in mathematics is not quite satisfactory (ASER, 2018; National Achievement Survey, 2017).

Lindquist (1997) shared the comments of high achievers in mathematics and concluded that many of them do not understand what they know. It may be said that some students whose achievement is good, also do not understand mathematics properly. Drawing from contemporary literature, Mann (2005) concluded that a high level of achievement in school mathematics is not a necessary ingredient for accomplishment in mathematics. In an environment where computation is the basis of assessment, high achievement is possible without mathematical understanding (Mayer & Hegarty, 1996). Stroop (1998) (cited in Schloglmann, 2009) discussed that some students did well in mathematics at school but they never really learned anything, because they rote learned everything. Bhatnagar (1968) reported that many students, though passed in examination, failed to achieve in terms of their abilities. Devlin (2000) states that current educational practices in elementary and secondary mathematics education focus on computation, formal reasoning, problem solving, they touch the applications but ignore the way of knowing and creativity. The knowledge acquired in such a learning process is very inflexible. That means, for instance, that when even just signs of the variables are changed, the learner cannot solve the exercise any more (Schloglmann, 2009). Needless to say that unless they understand concepts properly, they will not be able to apply its principles in problem solving and form an entire mapping of any concept in mind. These undeveloped concept-images limit the ability of mathematical applications in routine life as well as higher education and research.

The understanding of concepts is an important aspect of learning mathematics. Without understanding concepts in mathematics, mathematical success cannot be achieved and act as an epistemological obstacle for further studies. According to Schloglmann (2009), "Understanding mathematics refers to the essence of mathematics, which is to develop concepts and to use signs in a specific way. To develop students' understanding of essential concepts like numbers, function or equations is a central duty of mathematics education at school."

Thus, concept attainment is a major goal of mathematics teaching. It is a cognitive process where a concept transforms from being a process to become a mental object. Concept has been defined by Bruner et al. (1956) as, "a class or grouping of response, an act of categorisation, of 'rendering equivalent'. The act of categorisation involves rendering discriminately different things equivalent, to group the objects and events and people around us into classes, to respond to them in terms of their class membership rather than uniqueness."

The nature of mathematical concepts is abstract, so mastery over concept is a tough task. There are many studies conducted in the field of conceptual understanding and researchers have suggested that traditional method of teaching is not very effective for conceptual understanding. The studies suggested for the use of the concept attainment teaching model for teaching of mathematical concepts (Lekha, 2000; Prabhakaran & Rao, 1998; Minikutty, 2005). Bloom (1976) projected that 50 per cent of variance in school learning may be explained by cognitive characteristics. It is important to know the variables which are accountable for growth of attainment of concepts.

Nous and Raven (1973) and Ring and Novak (1971) (cited in Pandey, 1981) have shown that cognitive style is a factor which should be considered in concept attainment. Cognitive domain is divided mainly into two parts— first is intellect and second is creativity. Carlton (1959) also reported difference in types of mathematical mind, one that is logical and the other one is intuitive. Pehkonen (1997) discussed the balance between logic and creativity. Krutetskii (1969) refers to children who are gifted in mathematics, i.e., having "a mathematical frame of mind". Mann (2005) emphasizes that a balance between left and right hemisphere is needed for mathematical accomplishment. He further said that yet many students leave school with undeveloped right side of brain. This right side of brain is accounted for creativity. A balanced development of both sides of the mind is necessary for mathematical accomplishments.

Drefyus and Eisenberg (1996) said that a good mathematical mind is capable of flexible thought and can manipulate and investigate a problem from many different aspects. It is not uncommon to hear a student explain their poor performance in mathematics as being due to lack of mathematical mind (Aiken, 1973). Stadler (2004) considered the ability to create an internal picture of an abstract mathematical concept (Concept attainment) to be a major part of mathematical ability. Thus a question arises, does mathematical mind contribute in mathematical ability? If does, then what is the extent of this contribution? There are two facets of mathematical mind. One is formal or logical and other is intuitive or creative. Some studies revealed that most secondary students do not acquire formal level of cognitive development (Pandey, 1987; Rai, 1989). Is poor logical thinking ability accountable for difficulties in mathematical concepts? The answer of this question is important for mathematical progress. Mathematical creativity is the ability to think divergently in solving mathematical problems and produce unique solution. Can this ability help in the attainment of mathematical concepts? The study about this question would help the teachers to select appropriate strategy for teaching concepts.

Studies conducted in the field of mathematics are basically centralized on achievement. Though few researches were conducted in the direction of conceptual understanding, the confusion still exists about the contribution of important cognitive and non-cognitive variables towards the concept attainment.

Studies also report that several non-cognitive factors are accountable for achievement (Bloom, 1976; Singh, 1976). Vattno (1987) reported that only 50–70 per cent of total factor variance could be accounted for by all measurable intellectual factors, where as 30–50 per cent of the total factor variance is attributed for non-intellectual factors. A large number of studies reported that family background and socio-economic status (SES) is an important factor for achievement (Comber and Keeves, 1973; PISA, 2009; White, 1982). But the contribution of SES in attainment of mathematical concepts is not very clear. The present study attempts to find the contribution of logical thinking, mathematical creativity, socio-economic status and concept attainment model on the attainment of concepts in mathematics.

Concept

Concept is defined as a generalised idea about a class of objects, events, ideas, processes and relations on the basis of its common essential attributes.

Attainment of Concepts

Attainment of concepts refers to a clear mapping of essential and non-essential attributes of concept, seeing difference between examples of concepts drawn on the basis of essential and non-essential attributes, producing examples and non-examples, establishing relation with other concepts and identifying its supra-ordinate, co-ordinate and sub-ordinate concepts and differentiating them with the concepts.

Logical Thinking

Logical thinking is a process in which one uses thoughtful reasoning consistently in solving problems, framing inferences and making conclusions. The basis of logical thinking is sequential thought in situations that involves thinking for constructs and relationships in specific context. This process involves taking the ideas, facts and conclusions involved in a problem and arranging them in a sequential progression. Logical thinking is meaning-making procedure leads to a deeper understanding by presenting arguments that is an organised and disciplined way of convergent thinking. Logical thinking is a systematic, rational and convergent in nature, is often referred to as analytical thinking (Hicks, 1991).

Mathematical Creativity

Mathematical creativity is ability to think divergently in solving mathematical problems and produce unique solution in one's own way. It breaks the mental set and helps to solve problem by exploring the areas outside the individual's known content universe involving imagination and intuition. Singh (1988) stated that, "Mathematical creativity is the ability to produce unusual or original concepts, theorems, principles and unique method for problem solving in mathematics. It is the process of generating significant ideas, making theoretical ideas practical, converting innovative ideas of other fields into the new field."

Objective of the Study

To find out the contribution of logical thinking ability, mathematical creativity, socio–economic status and teaching through concept attainment model on attainment of concepts in mathematics.

Null Hypothesis [Ho]

There is no significant contribution of logical thinking ability, mathematical creativity, socio-economic status and teaching through concept attainment model on attainment of concepts in mathematics.

Method

Population and Sample

The population of present study was Class IX mathematics students of U.P. In present study cluster sampling technique was used. At first stage, six districts were selected randomly from all districts of population. In second stage, from each district three secondary schools of U.P. board were selected and then one section of Class IX of mathematics was selected randomly from each selected schools. All students of these sections were included in the sample. In statistical analysis 524 students were considered as sample, who participated in all tests and treatments.

Tools and Treatments

The following tools were used in the study.

1. **Mathematical Concept Attainment Test (MCAT):** For measuring concept attainment in mathematics MCAT was developed by researcher for selected six concepts (natural number, whole number, integer, rational number, irrational number and real number). MCAT was developed on the basis of Conceptual Learning and Development Model developed by Klausmeier and Hooper (1974) for concept attainment and Classroom Concept Learning Schema of Frayer, Fredrick and Klausmeier (1969), after concept analysis for identification of elements of concepts.
2. **Lesson Plan:** Lesson plans were developed by researcher for selected six concepts of Mathematics, based on concept attainment model, which were developed by Joyce and Weil (1986) based on theory of concept attainment of Bruner, Goodnow and Austin (1956).
3. **Logical Thinking Test:** This test was developed by Longeot (1965) for measuring operational reasoning of students. This test has been developed keeping in view the real sprit of piagetion concept of developmental stages of logical thinking. It measures four types (Class inclusion, propositional, proportional and combinatorial) of operational reasoning. Pandey and Bhattacharya (1985) adapted it in hindi named *Tarkik Chintan Parikshan.*
4. **Mathematical Creativity Test:** Mathematical creativity test has been developed by Singh (1985) to identify mathematical talent of secondary level age group students. Test is based on comprehensive set of criteria developed by Balka (1974) for measuring creative ability.
5. **Socio-Economic Status Scale:** For measuring the socio-economics status of Class IX students Upadhyay and Saxena's (2008) socio economic status scale was used. The scale consists of 31 items related to— (i) Personal information (ii) Family (iii) Education (v) Income and (v) Others Cultural and Material possessions).

To predict the attainment of concepts in mathematics and study the contribution of logical thinking ability, mathematical creativity, socio-economic status and teaching through concept attainment model on attainment of concepts in mathematics, step-wise multiple regression analysis was carried out. Scores of students on logical thinking, mathematical creativity and socio-economic status were found through corresponding tests. The concept attainment model is like a treatment, so no direct score can be found on this variable

After conduction of pre-test of MCAT, treatment of concept attainment model as a teaching method was employed on students. After teaching through this model post-test of MCAT was administered and the difference between pre-test and post-test scores for each student was considered as a gain score of students. This gain score for each student was considered as score on concept attainment model because apart from teaching through CAM no other independent and extraneous variables are accountable for difference between means of pre tests and post-tests scores. For testing the significance of difference between means of pre-tests and post-test scores on MCAT t-test was used.

Results

Results show a significant difference in the pre-test and post-test scores (Mean pre-test score= 22.08, post-test= 30.91, N= 524, t=90.96, $p<0.01$) on MCAT. It thus can be assumed that the gain in post-test score is the result of teaching through concept attainment model (CAM). The gain score was considered as a score on CAM for regression analysis. As the correlation between pre-test and post-test score was very high (0.905), post-test scores instead of pre-test scores was considered as score on attainment of concepts in mathematics as a dependent variable for the purpose of regression analysis. The results of regression analysis are shown in Tables 1, 2 and 3.

Table 1

Multiple Correlation Coefficient and Coefficient of Multiple Determination between Predictors and MCAT

S.N.	Independent Variable or Predictor	Multiple Correlation Coefficient (R)	Coefficient of Multiple Determination (R2)
1.	(Constant), CAM	0.602	0.362
2.	(Constant), CAM, Logical Thinking	0.757	0.573

3.	(Constant), CAM, Logical Thinking, Mathematical Creativity	0.763	0.583
4.	(Constant), CAM, Logical Thinking, Mathematical Creativity, SES	0.767	0.585

To study the significance of contribution of independent variables on dependent variable analysis of variance was used, which are shown in the Table 2.

Table 2

Analysis of Variance for Study the Significance of Contribution of Independent Variables on Attainment of Concepts in Mathematics

S. No.	Independent Variables or Predictors	Variance	Sum of Squares	*df*	Mean Square	*F*	*p*
1.	(Constant), CAM	Regression Residual Total	4956.775 8742.697 13699.471	1 522 523	4956.775 16.748	295.95	<0.01
2.	(Constant), CAM, Logical Thinking	Regression Residual Total	7845.818 5853.653 13699.471	2 521 523	3922.909 11.235	349.15	<0.01
3.	(Constant), CAM, Logical Thinking, Mathematical Creativity	Regression Residual Total	7982.380 5717.091 13699.471	3 520 523	2660.793 10.994	242.013	<0.01
4.	(Constant), CAM, Logical Thinking, Mathematical Creativity, SES	Regression Residual Total	8054.843 5644.628 13699.471	4 519 523	2013.711 10.876	185.152	<0.01

For study the contribution of independent variables on attainment of concepts in mathematics, step-wise multiple regression analysis was used and Table 3 contains the results.

Table 3

Stepwise Multiple Regression Analysis for Study the Contribution of Independent Variables on Attainment of Concepts in Mathematics

Steps	Independent Variables or Predictor	Regression Coefficient (B)	Constant	Multiple Correlation Coefficient (R)	Coefficient of Multiple Determination (R2)
1.	CAM	1.383	18.694	0.602	0.362
2.	CAM, Logical Thinking	1.274 0.438	10.009	0.757	0.573
3.	CAM, Logical Thinking, Mathematical Creativity	1.265 0.367 0.028	7.424	0.763	0.583
4.	CAM, Logical Thinking, Mathematical Creativity, SES	1.266 0.354 0.026 0.030	6.294	0.767	0.585

From Table 3 it is clear that 58.5 per cent of total variances of scores on attainment of concepts in mathematics was explained by teaching through concept attainment model, logical thinking, mathematical creativity and SES. Further, it is obvious that individual contributions of concept attainment model (CAM) is 36.2 per cent, logical thinking ability is 21.1 per cent, mathematical creativity is 1 per cent and individual contribution of SES is 0.2 per cent to the attainment of concepts in mathematics. Therefore, the null hypothesis that there is no significant contribution of logical thinking ability, mathematical creativity, socio–economic status and teaching through concept attainment model on attainment of concepts in mathematics is rejected and alternative hypothesis that there is a significant contribution of logical thinking ability, mathematical creativity, socio–economic status and teaching through concept attainment model on attainment of concepts in mathematics is accepted.

On the basis of the results of the study, regression equation for prediction of the attainment of concepts in mathematics can be written as follows — Attainment of Concepts in Mathematics = 6.294 + 1.266 (Gain Score from teaching through Concept Attainment Model) + 0.354 (Score on Logical Thinking) + 0.026 (Score on Mathematical Creativity) + 0.036 (Score on Socio-Economic Status). With the help of this regression equation attainment of mathematical concepts will be predicted on the basis of teaching through CAM, logical thinking ability, mathematical creativity and SES. This equation shows the predictive efficiency of CAM, logical thinking, mathematical creativity and SES on attainment of concepts in mathematics.

Discussion

In present study the contribution of concept attainment model (CAM), logical thinking, mathematical creativity and SES on concept attainment in mathematics were found significant. CAM (score gained after teaching through CAM) explains 36.2 per cent variance of concept attainment in mathematics. The contribution of CAM on attainment of mathematical concepts is obvious. Many researchers reported that CAM is a very effective model for conceptual attainment and understanding (e.g., Minikutty, 2005; Prabhakaram & Rao, 1998). Logical thinking explains 21.1 per cent variance of concept attainment in mathematics. For attainment of mathematical abstract concepts, logical thinking abilities are very important ingredient. Some researchers (Cantu & Herron, 1978; Lawson & Karplus, 1977) pointed out that formal reasoning abilities are highly correlated with understanding of concepts at different levels of abstraction. Schloglmann (2009) identified the problems with mathematics as being difficulties with logical thinking. Therefore, logical thinking and concept attainment teaching model would play important role in attainment of mathematical concepts. It is notable that common variance between CAM (gain score) and logical thinking is only 1.06 per cent. Bither (1991) found that formal operational reasoning explains 29 per cent variance of mathematical achievement. Pandey (1987) found that logical thinking covers 64 per cent variance of concept attainment in physics. Rai (1989) found that logical reasoning covers 29 per cent variance of conceptual understanding in physics.

Mathematical creativity explains one per cent variance of attainment of mathematical concepts in mathematics; even then this contribution is significant. This may be explained in the light of fact that the majority of the variance of mathematical creativity was accounted by logical thinking, because correlation between mathematical creativity and logical thinking is 0.603 and common variance among them is 36.36 per cent. The partial correlation between mathematical creativity and logical reasoning after controlling SES and gain score was also found 0.577, which indicates that mathematical creativity and logical reasoning are really correlated with each other and they shared common variances of attainment of mathematical concepts. The same is the case of SES, where SES covers only 0.2 per cent variance of attainment of mathematical concepts; even then this contribution is significant. The common variance among SES and logical thinking is 6.05 per cent and among SES and mathematical creativity is 4.7 per cent measured, hence it can be said that majority of variance of SES was accounted by logical thinking and mathematical creativity, that's why the contribution of SESS on concept attainment in mathematics is too less.

Therefore, a broad inference can be drawn that attainment of concepts in mathematics is not only the result of teaching with concept attainment model but also the result of both facets of mathematical mind, i.e., logical thinking and mathematical creativity and non-cognitive factor such as SES. However, remaining 41.5 per cent of the variance is unaccounted for. That may be the result of other factors which the researcher could not consider due to time constraint.

The present study produces the predictive equation for attainment of concepts in mathematics. The equation may be utilized for teachers in predicting attainment of concepts in mathematics for IX-graders. This regression equation can help the teachers for selection of students in mathematics and predicting their attainment and other purposes related to mathematics learning. The result of this study establishes that concept attainment model, logical thinking, mathematical creativity, and socio-economic status contribute in attainment of mathematical concepts significantly. Findings of present study suggest that teachers should use concept attainment model for teaching mathematics and teachers should also be trained in Piagetian theory as it applies to instruction and helps the students to acquire formal thinking. Students should be trained to explore their creative ability and use divergent thinking in mathematical problem solving. The present study also indicates that socio-economic status contributes in attainment of mathematical concepts. Equal educational facilities are necessary for development of human resource through mathematical accomplishment.

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