Pre-service Teachers' Beliefs Concerning the Nature of Mathematics and Teaching-learning of Mathematics

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ABSTRACT

The National Curriculum Framework—2005 calls for an epistemological shift in school mathematics, advocating a shift from 'narrow' goals to 'higher' goals of mathematising. However, to translate this vision into reality, one needs to recognise the significance of the agency of teacher. This necessitates a fundamental reconceptualisation of teacher preparation and professional development programmes, wherein investigating and challenging the teachers' beliefs have a crucial role. This study explores pre-service teachers' beliefs about nature of mathematics and its pedagogy. The paper also attempts at discussing probable consequences of the beliefs held by them for teaching and learning of mathematics. Results show that structural spaces need to be created to engage prospective teachers in the process of reflection, which in turn can help critically inform practice.

Introduction

Philosophical Perspectives on Nature of Mathematics

Mathematics is generally viewed as 'a body of knowledge', and the aims of teaching-learning mathematics are generally thought of as 'to acquire the basic mathematical skills and numeracy and to solve practical problems with mathematics'. However, there are controversies over what mathematics is and what the aims of teaching-learning mathematics are (Ernest, 2014, p. 3). The academic philosophy of mathematics addresses technical philosophical issues in relation to the epistemology of mathematics,

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and the values and beliefs inherent to the development and validation of mathematical knowledge. This body of literature documents a number of different academic philosophies of mathematics: realism, Platonism, empiricism, logicism, intuitionism, formalism, conventionalism, and social constructivism.

The diverse philosophical stances about nature of mathematics hold different positions about mathematical knowledge. Some of these (realism, Platonism, formalism, and logicism) can be loosely grouped together and described as 'traditional' philosophies of mathematics because they share an absolutist view of mathematics. In such a view, mathematics is seen as a body of infallible and objective truth, super-human, free from social, cultural and political influences and thus absolute (Ernest, 2014) and mathematical activity is seen as highly abstract, formalised and decontextualised. Such a view assumes a separation between cognitive processes and the settings and activities of which they are a part and treats knowledge as a factual commodity or compendium of facts. At school level, this conception of mathematics gets manifested in the form of cultural discontinuity between 'academic' mathematics and everyday mathematics; resulting in the exclusion of some particular groups.

However, these absolutist conceptions have been challenged, and in the past sixty years a new philosophy of mathematics has been emerging which proposes a humanistic alternative, that is, fallibilism (Ernest, 2014). The fallibilist or 'new' philosophies of mathematics include philosophical stances about nature of mathematics such as fallibilism, quasi-empiricism, humanism, and social constructivism. The fallibilist position views mathematics as socially and culturally constructed; and politically situated (Ernest, 2014). This position redefines mathematics as a fallible social construction; a coming to know, continually expanding field of human creation and invention; and provides a rationale as well as a foundation for 'inclusive' approaches to mathematics; wherein the social contexts of the uses and practices of mathematics can no longer be legitimately pushed aside. Mathematics, then, needs to be studied in living contexts which are meaningful and relevant to its learners, including their languages, cultures and everyday lives, as well as their school based experiences.

Although the link between philosophy of mathematics and its teaching-learning is often invisible, some well-known mathematicians (for example, Reuben Hersh, 1979) have indicated that there exists a direct link between the two. However, the 'new' (fallibilist) view of mathematics remains a controversial philosophy of mathematics and has fewer supporters than those of absolutism Ernest, 2014). As mathematics educators, we feel that mathematics teachers need to be aware of the philosophical debates and disputes regarding the nature of mathematics. They need to be aware of and acknowledge the legitimacy of both the fallibilist and absolutist views of the nature of mathematics.

The Indian Context

Keeping in tune with the theoretical advancements in the understanding of philosophy, history and sociology of mathematics there has been a review of the purposes of mathematics education at the national level. The *National Curriculum Framework-2005* advocates a shift from achieving 'narrow' goals to 'higher' goals of 'mathematising'; a shift in focus from mathematical content to mathematical learning environments, offering multiplicity of approaches, procedures and solutions (NCERT, 2006). The shift from the conventional noun 'mathematics' to the verb 'mathematising' poses a challenge to the conventional epistemology of mathematics.

Teachers' Beliefs about Mathematics and its Teaching-learning

The prospective teachers' limited knowledge of a particular teaching subject is often discussed in the academic discourse, but discussions about beliefs that prospective teachers carry about the nature of discipline are seldom thought about (Sullivan, 2003). A teachers' way of teaching and changing her/his practice of teaching, in fact, is affected by several factors—teachers' beliefs, expectations, experience, pedagogical and content knowledge, certification and licensure, and educational attainment that constitute a *teacher's background* (Goe, *et al., 2008*). Of all these *inputs* that a teacher brings to her/his position, belief is a major one as it also influences teachers' planning, decision-making and subsequent classroom behaviour.

Researches also attest to the fact that teaching practices in a particular subject are affected by the beliefs teachers hold about teaching-learning of and the nature of that discipline. Teachers' beliefs about the nature of mathematics and its teaching-learning have been found to influence their teaching practices in mathematics (Ernest, 1989; Emenaker, 1996; Beswick, 2007; Dede and Uysal, 2012). Thus, it is critical to address the beliefs pre-service teachers hold toward mathematics since these beliefs can have a strong influence on their approaches to teach mathematics (Emenaker, 1996) and 'in turn (those teacher actions) have a tremendous impact on students' belief systems' (Raymond, Santos, and Mansingila, as cited in Emenaker, 1996).

Beliefs are basic knowledge and general understandings which a person holds. Although researches on teachers' beliefs have increased in the recent decades the theoretical construct of belief lacks a commonly agreed definition (Dede and Uysal, 2012; Leder *et al.*, 2002). The term 'belief' has been considered equal to 'personal judgments', concepts, meanings, propositions, rules, preferences, 'mental constructs', and 'psychologically held understandings, premises, or propositions about the world' (Dede and Uysal, 2012). In general terms, beliefs have been defined as the lenses through which an individual makes sense of the world around and as such influence the way one interacts with the world (Philipp, as cited in Swars, 2007a). This implies that teachers' beliefs are central to what occurs in classrooms. Visible teaching practices that occur in the classroom are partly a result of the hidden interpretive lenses a teacher holds (Aydin *et al.,* 2010).

Given the multiplicity of definitions of 'beliefs', we have chosen the one given by Raymond (1997), for the purposes of the present study, which defines mathematics beliefs as 'personal judgments about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics' (*ibid.* p. 552).

Attempts have also been made to categorise mathematics beliefs, and frameworks for studying the mathematical beliefs have been proposed. For example, Dionne, 1984 (as cited in Torner and Pehkonen, 1999) has proposed the following perspectives of a mathematical belief system:

- Mathematics is seen as a set of skills (*traditional perspective*): Doing mathematics is doing calculations, using rules, procedures and formulas.
- Mathematics is seen as logic and rigor (*formalist perspective*): Doing mathematics is writing rigorous proofs, using a precise and rigorous language and using unifying concepts.

- Mathematics is seen as a constructive process (*constructivist perspective*): Doing mathematics is developing thought processes, building rules and formulas from reality experience and finding relations between different notions.

Ernest (1989) has also proposed similar views of mathematics:

- *The Instrumentalist View:* The view that mathematics is a useful but unrelated collection of facts, rules and skills.
- *The Platonist View:* The view of mathematics as a static but unified body of knowledge, consisting of interconnecting structures and truths. Mathematics is a monolith, a static immutable product, which is discovered, not created.
- *The Problem Solving View:* The dynamic, problem-driven view of mathematics as a continually expanding field of human inquiry. Mathematics is not a finished product, and its results remain open to revision.

The above mentioned views correspond more or less to Dionne's three perspectives of a mathematical belief system (Torner and Pehkonen, 1999).

Taking into cognisance new epistemology of mathematics, NCF-2005 acknowledges the 'cultural grounding of mathematics' when it notes that "mathematical competence is situated and shaped by the social situations and the activities in which learning occurs. Hence, school mathematics has to be in close relation to the social worlds of children where they are engaged in mathematical activities as a part of daily life" (NCERT, 2006, p. 11). Such a conception of mathematics necessitates a fundamental reconstruction of school mathematics at all levels — curricular choices, pedagogy, assessment, etc. The shift envisaged, poses fundamental changes in teacher preparation and professional development. As the NCF (2005) also points out, "more so than any other content discipline, mathematics education relies very heavily on the preparation that the teacher has, in her own understanding of mathematics, of the nature of mathematics, and in her bag of pedagogic techniques" (p. 6), it becomes pertinent to examine teachers' beliefs about mathematics.

The present study, therefore, aimed at investigating the beliefs of pre-service teachers about the nature of mathematics, and teaching-learning of mathematics, in the light of *National Curriculum Framework-2005*.

Method

Participants

The sample consisted of 50 pre-service elementary teachers enrolled in a four year undergraduate teacher education programme offered by the University of Delhi. All the participants were in the first year of their course.

Procedure

A mathematics belief inventory consisting of 4-point Likert type items was used. Items were taken from Evans (2003) and Hart (2002), as adapted by Zakaria and Musiran (2010). The inventory consisted of 23 items covering three dimensions: *beliefs about the nature of mathematics*, *beliefs about teaching mathematics* and *beliefs about learning mathematics.*

The second phase of the study involved focused group discussion with the prospective teachers to further probe reasons for agreements and disagreements to the given statements, and to understand inconsistencies in their responses.

Results and Discussion

Beliefs about Nature of Mathematics

Conception of the nature of mathematics pertains to 'a teacher's belief system concerning the nature of mathematics as a whole' (Ernest, 1989). The pre-service teachers' views regarding the nature of mathematics were studied with the help of seven statements included in the rating scale which asked them to indicate the extent to which they agree/disagree with the statements. Table 1 summarises their responses.

Table 1

Beliefs about Nature of Mathematics

Item No.	Items	Strongly Disagree (%)	Disagree (%)	Agree (%)	Strongly Agree (%)
#3	Mathematics problems can be done correctly in only one way.	50.0	43.0	7.0	0.0
#7	Males are better at math than females.	66.0	17.0	10.0	7.0
#10	Some students have a natural talent for math and others do not.	7.0	35.0	47.0	11.0

#12	Mathematical knowledge is fixed and absolute.	12.0	11.0	49.0	28.0
#14	Mathematics is about reasoning, posing and solving problems.	2.0	22.0	49.0	27.0
#19	In mathematics something is either right or it is wrong.	4.0	18.0	58.0	20.0
#20	Some people are good at mathematics and some are not.	5.0	12.0	53.0	30.0

As shown in Table 1, majority (93%) of the respondents believed that there can be multiple ways of doing mathematical problems correctly, as against the common notion of one 'right' way to obtain one 'right' solution. This is further corroborated by a huge agreement (76%) to the statement that 'mathematics is about reasoning, and posing and solving problems', as against the popular perception of the discipline being dry and dull. However, majority (78%) of them also believed that 'in mathematics something is either right or it is wrong'— there cannot be any partial solution to a mathematical problem— either you know it or not; thus leading to anxiety (NCF, 2005). Also, most of the prospective teachers (78%) view mathematical truths as fixed and absolute, and thus fail to challenge the infallibility of mathematics and do not take into cognisance the new philosophy and epistemology of mathematics.

A large proportion of prospective teachers did disapprove that 'males are better at math than females' (83%). This shows a positive attitude towards girls' capabilities of doing mathematics and hold that mathematical abilities are not the monopoly of any one particular group. Still, most of them believed that some students have a 'natural talent' for mathematics (58%) and 'some people are good at mathematics than others' (83%). Such statements majorly focus on socio-political dimensions of mathematics education. Many of the teachers' assumptions come from the attitudes and beliefs prevailing in wider social contexts. Teachers need to challenge erroneous assumptions that link success in mathematics to some special talent/innate ability that only a few possess, and thus deconstruct what mathematical 'ability' and 'achievement' constitutes *(not asked in the listed data).* This is essential for equitable mathematics instruction for all, as also envisaged by NCF-2005.

Beliefs about Teaching Mathematics

Beliefs about teaching mathematics pertain to the 'teacher's conception of the type and range of teaching actions and classroom activities contributing to her/his personal approaches to the teaching of mathematics. It includes mental imagery of prototypical classroom teaching and learning activities, as well as the principles underlying teaching orientations' (Ernest, 1989).

Table 2

Beliefs about Teaching Mathematics

Item No.	Items	Strongly Disagree (%)	Disagree (%)	Agree (%)	Strongly Agree (%)
#2	Mathematics should be taught as a collection of skills and algorithms.	0.0	5.0	81.0	14.0
#4	In mathematics, increased emphasis should be given to use of key words to determine which operation to use in problem solving.	5.0	20.0	56.0	19.0
#5	A major goal of mathematics instruction is to help student develop the belief that they have the power to control their own success in mathematics.	0.0	21.0	45.0	34.0
#8	More than one representation (picture, concrete material, and symbol set, etc.) should be used in teaching a math concept.	0.0	5.0	51.0	44.0
#11	Good math teachers show you the exact way to answer the question you will be tested on.	12.0	30.0	49.0	9.0
#13	In mathematics, skill in computation should precede word problems.	2.0	17.0	56.0	25.0
#15	Students should be encouraged to justify their solution, thinking and conjectures.	3.0	3.0	51.0	43.0
#17	Basic computational skills on the part of the teacher are sufficient for teaching mathematics.	10.0	54.0	32.0	4.0

To study the pre-service teachers' views regarding the teaching of mathematics, 8 statements were given in the rating scale which asked them to indicate the extent to which they agree/disagree with the statements. Table 2 presents a summary of their responses.

As shown in Table 2, majority (95%) of the respondents agreed on the desirability of using multiple representations (picture, concrete material, and symbol set, etc.) in teaching a math concept. These multiple modes of representation also correspond with Bruner's three modes of learning: *enactive, iconic,* and *symbolic*.

Majority of the prospective teachers also believed that 'students should be encouraged to justify their solution, thinking and conjectures' (94%) and 'mathematics instruction should aim at helping students develop the belief that they have the power to control their own success in mathematics' (79%), indicating trust in students' ability to construct mathematical knowledge and take control of their own learning.

Although the majority of respondents (64%) believed that only 'basic computational skills' on the part of the teacher are not sufficient for teaching mathematics, most of them (58%) felt that good math teachers should show their students the exact way to answer the question they will be tested on. On one hand they believed that computational skills constitute one aspect, not the only aspect of mathematics teaching, yet they are placing invariable emphasis on the procedural knowledge (that a teacher needs to impart) for being successful in mathematics. This kind of response contradicts with their earlier response that the students should be active enough in their learning and be given opportunities to hone their cognitive capacities of reasoning and thinking so that they may be in control of their own success in mathematics, as it continues to see the teacher as the epistemic authority who needs to show the 'exact way' of doing mathematics. Such a response also leaves unacknowledged the varied ways of thinking and solving that students bring to the classroom.

Also, computational fluency becomes a central concern of mathematics for the prospective teachers since majority of them (95%) believed that 'mathematics should be taught as a collection of skills and algorithms'. Such a belief will further mystify mathematics to the students and become a central cause of anxiety for them as when conceptual understanding, construction of knowledge are replaced by procedural fluency, and symbols are manipulated without understanding, dissociation from the discipline takes place (NCF-2005).

Beliefs about Learning Mathematics

Beliefs about learning mathematics pertain to 'the teacher's view of the process of learning mathematics, what behaviours and mental activities are involved on the part of the learner, and what constitute appropriate and prototypical learning activities. Thus, these involve aims, expectations, conceptions and images of learning activities and of the process of learning mathematics in general' (Ernest, 1989).

The pre-service teachers' views regarding the learning of mathematics were studied with the help of 5 statements included in the rating scale, which asked them to indicate the extent to which they agree/disagree with the statements. Table 3 summarises their responses.

Table 3

Beliefs about Learning Mathematics

Item No.	Items	Strongly Disagree (%)	Disagree (%)	Agree (%)	Strongly Agree (%)
#1	Mathematics should be learnt as sets of algorithms/rules that cover all possibilities.	5.0	21.0	65.0	9.0
#6	A demonstration of good reasoning should be regarded even more than student's ability to find correct answers.	5.0	32.0	39.0	24.0
#9	In mathematics, you can be creative and discover things by yourself.	7.0	12.0	61.0	20.0
#16	Learning mathematics must be an active process.	3.0	3.0	63.0	31.0
#18	To solve most math problems you have to be taught the correct procedure.	0.0	10.0	65.0	25.0

As shown in Table 3, most of the respondents (94%) agreed that mathematics learning must be an active process, and considered mathematics to be a creative endeavor where students can engage in discovering things on their own (81%). This belief is consistent with the aim of 'mathematisation of the child's thought processes'—the higher aim of mathematics teaching envisaged by the NCF-2005.

Although 63% agreed that more than student's ability to find correct answers, a demonstration of good reasoning should be regarded important; 90% respondents still believed that the correct procedure has to be taught to help students solve most math problems. Thus, again a contradiction is seen in the prospective teachers' beliefs: limiting their own belief about engaging the students' meta-cognitive abilities to the emphasis on being taught the procedural fluency.

Also most of the respondents (74%) agreed that 'mathematics should be learnt as sets of algorithms/rules that cover all possibilities'. Thus, the role of problem solving and active learning in mathematics was underplayed and the emphasis on gaining fluency in procedural knowledge overplayed. This belief again gets corroborated when 100% respondents agree that memorising formulas and procedures is important for the students to be good at school mathematics (Table 4).

Table 4

Teachers Belief about Students' Understanding of Mathematics

Item No.	Items	Not Important (%)	Important (%)	Very Important (%)
To be good at mathematics at school, how important do you think it is for students to:				
#21	Remember formulas and procedures?	0.0	44.0	56.0
#22	Think in a sequential manner?	23.0	64.0	13.0
#23	Be able to provide reasons to support their solutions?	12.0	30.0	58.0

Focus Group Discussion: Probing Further

The results showed that in their first year of the programme, prospective teachers' understanding related to constructivist and socio-constructivist theories was in the process of emerging. They became familiar with the notions of child-centered learning, modes of representations, discovery learning, etc. This was reflected in some of their responses. However, what needs attention are the myriad interpretations that prospective teachers have of many of these ideas. For instance, most of them agreed that mathematics is about solving problems. Varied interpretations for problem solving in mathematics classrooms were revealed during the discussion, such as solving arithmetic, integration and differentiation problems quickly and accurately; efficiently applying shortcuts and formulae (also considered essential for cracking competitive examinations); solving mathematical puzzles and riddles.

Also, when it comes to translating some of these ideas, such as the role of reasoning, activity, etc., in mathematics classrooms which most of them agreed to, what rules the discussion is 'teaching as they were taught'. In relation to mathematics pedagogy, their own school experiences, which have been instrumental in shaping their perceptions of the discipline, come to the foreground. The statements like: "Practice is must for success of a child in mathematics", "Doing mathematics is about solving more and more similar examples and questions", "Students need to be good at learning formulae and rules, and mental ability to solve problems" – predominated much of the discussion.

The inconsistencies in their responses could possibly be understood in the light of the complex interactions between the emerging understandings of the 'new' ideas (which are discussed as a part of teacher education programme they are enrolled in) as prospective teachers, and their own school experiences as learners of mathematics (caught in the cycle of teaching as they were taught).

Teacher education institutions, thus, need to create spaces where the student teachers can bring to fore these conflicts and contradictions among their ideas, reflect on the varied interpretations they have and their consequences on teaching and learning, and engage in a continual process of questioning their beliefs and images of the discipline vis-à-vis their past experiences and assumptions prevailing in the wider society regarding the discipline and its learners.

Conclusion

The findings of the study have serious implications for teacher education programmes as well as school education. Since 'change in beliefs is a crucial precursor to real change in teaching' (Swars, *et al.*, 2007b), it is imperative that pre-service teachers align their pedagogical beliefs with current thinking on teaching and learning mathematics in order to increase their efficacy for teaching mathematics. Thus, the teacher education programmes need to give opportunities to the prospective teachers, to reflect on their mathematics belief and practices from the very beginning of teacher preparation and continually do so. Such an early reflection on one's philosophy of teaching can help a teacher assess her teaching practice more honestly. Thus, the primary goal of mathematics teacher preparation should be to stimulate the examination and development of beliefs about mathematics and mathematics pedagogy. This may be the key to minimising inconsistency between beliefs and practice and improving the quality of mathematics instruction (Raymond, 1997).

Also, the inconsistencies between the vision of teacher education programmes, which is currently based on socio-constructivist principles, and the actual practices of teacher educators, which still follow the teacher centered approaches, do fail to influence the prospective teachers' belief in constructivist lines. Since prospective teachers are expected to use student centered methods, pedagogical practices that support constructivist theory can be nurtured by engaging such teachers in constructivist experiences both in learning mathematics and in teaching mathematics (Hart, 2002). This would, in turn, facilitate nurturing beliefs that are consistent with the current philosophy of learning and teaching as envisaged by the NCF-2005.

References

Aydin, M., et al. 2010. Mathematics Teacher Educators' Beliefs About Teacher Role. *Procedia Social and Behavioural Sciences*. Vol. 2, pp. 5468–5473.

Beswik, K. 2007. Teachers' Beliefs That Matter in Secondary Mathematics Classrooms. *Educational Studies in Mathematics*. Vol. 65. pp. 95–120.

Dede, Y. and F. Uysal. 2012. Examining Turkish Pre-service Elementary Teachers' Beliefs About the Nature and the Teaching of Mathematics. *International Journal of Humanities and Social Science*. Vol. 2. pp. 125-135.

Emenaker, C. 1996. A Problem-solving Based Mathematics Course and Elementary Teachers' Beliefs. *School Science and Mathematics*. Vol. 96. pp. 75–84.

Ernest, P. 1989. The Knowledge, Beliefs and Attitudes of the Mathematics Teacher: A Model. *Journal of Education for Teaching*. Vol. 15. pp. 13–33.

——. 2014. What is Mathematics and Why Learn It? In P. Andrews, and T. Rowland (Eds.), *Master class in Mathematics Education: International Perspective on Teaching and Learning*. Bloomsbury, London.

Goe, L., C. Bell and D.O. Little. 2008. *Approaches to Evaluating Teacher Effectiveness: A Research Synthesis*. National Comprehensive Center for Teacher Quality, Washington DC.

HART, L.C. 2002. Pre-service Teachers' Beliefs and Practice After Participating in an Integrated Content/Methods Course. *School Science and Mathematics*. Vol. 102. pp. 4–14.

LEDER, G.C., E. PEHKONEN AND G. TORNER (EDS.), 2002. *Beliefs: A Hidden Variable in Mathematics Education?* Kluwer Academic Publishers, Dordrecht.

NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING. 2005. *National Curriculum Framework-2005: National Focus Group on Teaching of Mathematics*. NCERT, New Delhi.

RAYMOND, A.M. 1997. Inconsistency Between a Beginning Elementary School Teacher's Mathematics Beliefs and Teaching Practice. *Journal for Research in Mathematics Education*. Vol. 28. pp. 550–576.

SULLIVAN, P. 2003. Editorial: Incorporating Knowledge of and Beliefs About, Mathematics into Teacher Education. *Journal of Mathematics Teacher Education*. Vol. 6. pp. 293–296.

SWARS, S.L. 2007. The development of Mathematics Beliefs of Elementary School Teachers. *Georgia Educational Researcher*. Retrieved from: http://coefaculty.valdosta.edu /lschmert/gera/current_issue.htm

SWARS, S., *ET AL*. 2007. A Longitudinal Study of Elementary Pre-service Teachers' Mathematical Beliefs and Content Knowledge. *School Science and Mathematics*. Vol. 107. pp. 325–335.

TORNER, G. AND E. PEHKONEN. 1999. Teachers' Beliefs on Mathematics Teaching-comparing Different Self-estimation Methods: A Case Study. Retrieved from: http://www.uni-duisburg.de/FB11/PROJECTS/MAVI.html

ZAKARIA, E. AND N. MUSIRAN. 2010. Beliefs About the Nature of Mathematics, Mathematics Teaching and Learning Among Trainee Teachers. *The Social Sciences*. Vol. 5. pp. 346–351.