Research Innovation

Effective Intervention to Enhance Understanding in Mathematics—Report of a Successful Experiment

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Abstract

Teaching the first year undergraduate students, the foundations of mathematics, is a challenge and could be quite unnerving to a novice. The first few lectures in the First Year B. Sc. class are very crucial and the way in which one handles this will set the tone for the whole year. There are several reasons for this: the fall in enrollment for pure sciences including Mathematics resulting in the poor quality of students opting for Mathematics, their differing needs, and the disparate levels of language skills and mathematical capabilities. But a good number of these students end up teaching Science/ Mathematics in high school / higher secondary school and some of them do post graduation and teach in undergraduate colleges and engineering colleges too. So it is imperative that we target this group and try and motivate them, and teach them. Mathematics in a more effective manner. This is a report of a successful intervention since June 2006, using certain strategies to tackle these issues and enhance Mathematics learning, and the understanding of average learners of Mathematics.

Introduction

Origin of the Research Problem

Teaching the first year undergraduate students, the foundations of Mathematics, is quite a challenge and could be quite unnerving to the uninitiated. Many of the students are very disillusioned as they have not made it to professional courses like engineering, medicine or even the courses like B.Sc. IT or Computer Science because of poor scores in the examinations or for lack of funds or both.

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There is a major shift in emphasis from pre-college Mathematics to Undergraduate Mathematics, especially in subjects like Algebra, Real Analysis and Topology. The nature of the subject being what it is, it requires application, hard work and a certain level of mathematical maturity—the maturity to understand and articulate mathematical ideas and learn methods not only of problem-solving but of writing mathematical proofs. In the first year B.Sc. course around 120 - 150 students are admitted in the Physics- Mathematics- Chemistry and Physics–Mathematics-Computer Science subject combinations in our college. By the end of the first semester, more than 50 per cent of the students leave for Engineering, Management Studies and other courses. Among those who continue with B.Sc. 25-30 students opt for Mathematics as one of their subjects in the second year B. Sc; we are trying to teach such a group of students, principles of cardinality, countability, the infinite and the infinitesimal, and convergence and completeness.

Table I

Year	No. of Students Who Appeared for Mumbai University Final B. Sc Exam in Mathematics	Pass %
2001	1200	73
2010	800	55

The dwindling numbers and poor quality of students opting to major in Mathematics, their differing needs that derive from their family backgrounds and poor language skills complicate the matter further. In the author's experience as a mathematics teacher and as an examiner for the University Examination for several years, it has been observed that this experience is not unique to our college. 1200 students appeared for the third year University Examination in Mathematics in the year 2001 and 73 per cent of them passed. By 2010, the figures have dropped alarmingly to around 800 with only 55 per cent of them passing (See Table I); it is further seen that more students fail in abstract papers like Algebra or Analysis where they are not able to write a proof or convey ideas with clarity. The **more serious concern here** is that a large number of these students end up teaching Mathematics in high schools, higher secondary schools, undergraduate colleges and engineering colleges. So it is imperative that we target this group and try and motivate them, and teach them Mathematics in a more effective manner.

Epistemological Concerns

The nature of Higher Mathematics is such that to learn Mathematics one needs to have fluency with logic, notation, manipulation of symbols, and the ability to understand and articulate abstract ideas. According to S. G. Krantz, "As the subject develops, so do the skill sets needed to master the new ideas. The techniques needed to learn Calculus and solve problems in Calculus are different from the skills needed to learn abstract algebra, prove theorems about it. ..... Calculus requires problem solving skills and Algebra needs serious logic, axiomatic, set theory, sophisticated ability with proofs, and considerable tenacity. There is a marked qualitative difference in the two skill sets. The latter skill set is what mathematical maturity is all about". He clarifies further "mathematical maturity consists of the ability to handle increasingly abstract ideas, generalise from specific examples to broad concepts, formulate problems, work with analytical, algebraic, and geometrical concepts, and to move from the intuitive to the rigorous". To make this transition, the language proficiency in the medium of instruction (English) and proficiency in Mathematical language are very essential; otherwise it can hinder progress by creating bottlenecks to learning. Communication in Mathematics includes teaching-learning, writing, answering questions in class or examinations, and solving problems and justifying steps. Field medalist William P. Thurston wrote, in his article "On proof and progress in Mathematics", "our linguistic facility is an important tool for thinking, not just for communication. The language that we use reflects the level and the profundity of our thinking."

Pedagogical Concerns

Being good at Mathematics is not enough to teach Mathematics well. To teach Mathematics well, one should be able to conjecture, recognise patterns and generalise, teach students problem solving skills, be tenacious in pursuing problems, find examples to make a precise mathematical point, choose strategic examples and counter-examples, anticipate possible misconceptions, analyse the source of student error and use them to clarify concepts, and teach them the use of mathematical notation and language, and critique its use. The teacher's knowledge of Mathematics is crucial to their capacity to make judgments about planning and organising the content, to use teaching materials effectively, and to dynamically engage students to enhance their learning. The

popular but erroneous notion is that Mathematics teachers merely solve problems. They must explain concepts, theories, listen, justify, evaluate assignments, and assess students' understanding. In short, teachers need skills and knowledge in order to teach effectively. All these tasks involve both mathematical reasoning and pedagogical thinking. According to Hyman Bass, Professor of Mathematics and Mathematics Educator, *the knowledge, practices, and habits of mind of research mathematicians are not only relevant to mathematics education, but that this mathematical sensibility and perspective is essential for maintaining the mathematical balance and integrity of the educational process.*

Given the nature of mathematics, the task of making classroom teaching of mathematics effective and inclusive is indeed a challenge. The author initiated certain intervention since 2005-06 based on demystification approaches, learner centric methods and error analysis with the immediate goal of reversing the downward trend in passing percentage and the larger goal of fostering effective and inclusive teaching in mind.

Intervention

The first year students are a disillusioned lot with muddled understanding of concepts, faulty logic and a lack of communication skills. We are trying to teach such a group of students, abstract concepts and axiomatic. The result could be disastrous. The erosion in the quality, over the years, of students enrolling for Mathematics, had a cascading effect. In 2005, the pass percentage of third tear B. Sc mathematics students touched an all time low of 53 per cent, 9 per cent lower than the universities pass percentage. It called for drastic measures of intervention. The author initiated certain activities based on demystification approaches, reinforcement through drills, math communication skills, and more effective teaching strategies.

Demystification Approach

As part of these efforts, a credit course in Mathematics was initiated in the year 2005-06. A theme was chosen for each year. Some of the themes chosen have been *Numbers, Geometries, A Mathematician of your choice, Cryptography,* and *Infinite Series.* Popular lectures on the theme by experts are organised in the beginning of the year. Then all students, in groups of not more than three, select a topic, with the help of teachers, and submit an assignment which included a bibliography too. Students were

encouraged to give presentations and book reviews. Credits were earned through these assignments, seminars, book reviews, quality contribution to the Math Window magazine and also consistently good academic performance in Mathematics in the examinations in all the three years. The other activities included screening of movies on Mathematics, literary sessions where excerpts from books on Mathematics or Mathematicians are read, audio-visual lectures on topics like Computer animation, the Golden Ratio, attending public lectures on Mathematics and so on. The literary sessions, popular lectures and book reviews were aimed at improving their general language proficiency and to arouse interest in Science and in particular Mathematics.

Mathematics Communication Skills

It is observed in the first year of the B.Sc programme that a majority of the students have been learning mathematics by literally scanning the text like an image or learning by rote. On asking a student to read or write a proof or a definition, the student would halt invariably when notations enter the statement. As a result of the inability to understand the logical connections and/or the inability to comprehend the notation, many students are not able to recall definitions and proofs completely. So the students were given drills in mathematics communication-drills in notation and logic, drills in reading/writing a mathematical statement; their erroneous assumptions and arguments were discussed. Learning takes place at various levels—visual, symbolic, and verbal. So students are given exercises in converting verbal representations to symbolic and vice versa. This exercise continues for a while explicitly and then, in a more subtle way, throughout the three-year degree programme to help them gain understanding and the ability to articulate ideas.

Effective Teaching Strategies

At the next level, more emphasis was laid on learner-centric pedagogies, error analysis, and a problem-solving approach. Engaging students in problem solving (on the board or in groups), discussing their misconceptions and faulty arguments which gives them real time feedback, getting them to recall, precisely, statements of theorems and definitions, asking them to justify a particular step in a proof were all part of these strategies. Conceptual understanding and articulation, and an ability to see the overall picture are also equally important. So, as they gain more mathematical maturity,

the students were asked to rephrase definitions, fill gaps in proofs, negate mathematical statements, write converse of mathematical statements, summarise the various steps in a proof, maintaining journals and so on. By using these strategies, the students were made to think aloud by analysing and vocalising their thoughts which in turn helped them clarify their thought process. In short, they were trained to write mathematics to learn mathematics.

On the other hand, the teaching plans were discussed, and the ideas and practices were systematically examined to improve their effectiveness with the students, by interacting with colleagues and using other resources. Every year while discussing teaching plans for various topics, it was deliberated, for example, whether to teach the proofs of theorems like *the cardinality of a set is strictly less than that of its power set* or *the existence and uniqueness of the determinant function*, for a particular first or second year batch; how best to prove *the interval (0, 1) is uncountable or equivalent to the set of real numbers*. The teaching plans and the techniques were thus calibrated to suit the level of each batch and then efforts were made to raise the bar.

Discussion of Outcomes

A definite qualitative change is seen in the students as they progress from the first year to the third year. This is also reflected in the marks they score. At the entry level, the average marks are approximately 45 per cent and at the third year B.Sc. University examination, around 55-60 per cent of our students score a first class. There is also a marked improvement in the pass percentage. A comparison of the pass percentage of our students and the corresponding figures for the university is given in Table II.

It was possible not only to arrest the downward trend but also to improve the pass percentage far above the overall pass percentage of the university. It has also created the awareness among the faculty the need to create a classroom culture wherein the students feel free to interact, try to solve problems on the board, write journals. This model can be used for effective and inclusive teaching of Mathematics wherein the students are actively participating in the classroom, thereby leading to enhanced understanding of concepts.

Conclusion

The lesson that can be learned from the last six years is that we need to revamp the undergraduate Mathematics Programme in order

Table II

Year	Pass % in Mathematics in	
	University of Mumbai	Jai Hind College
2003-04	68	65
2004-05	64	53
2005-06	62	100
2006-07	51	86
2007-08	53	84
2008-09	53	74
2009-10	54	100
2010-11	55	94

to attract more and better students for teaching and for research. In order to achieve this we need to address the issues at many levels : Mathematical Communication Skills needs to be made a part of the first year curriculum through which the students can be trained in skills of comprehension, logic and communication of mathematical ideas; besides curriculum change one needs to relook at tertiary pedagogical practices too. Regular workshops of short duration on effective and inclusive practices of teaching mathematics may be organised for undergraduate teachers where there could be collaboration between mathematics teachers and mathematics educators. Finally, there is a need for more collaboration between the research mathematicians and the undergraduate teachers as this can help Mathematics teachers gain valuable knowledge and fresh perspectives, there by help improve the quality of Mathematics Education.

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