Convenient Rays for Locating Image formed by a Spherical Mirror using Ray Tracing Technique and Verification of Mirror Formula by Graphical Method

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Abstract

In this paper, the convenient rays for locating image formed by spherical mirrors using ray tracing technique are discussed. Only concave mirror is taken for the purpose of illustration and simplification. Ray tracing technique is used to locate images formed by concave mirrors using graphical method. The mirror formula is verified empirically using graphical method by measuring the values of image distance, object distance and focal length of the concave mirror. This technique of finding the image formed by a concave mirror using ray tracing technique by graphical method was used in teacher training programmes and it is being published for wider dissemination.

Introduction

Normally an image formed by spherical mirrors (concave and convex mirror) is taught at the secondary stage in the country. Ray tracing technique is used to find images formed by spherical mirrors. The paper tries to arrive at some of the convenient rays for locating images formed by spherical mirror using ray tracing technique. Preliminary concept of mirror formula is also introduced at secondary stage without giving derivation of the formula. Students are expected to use the mirror formula while solving problems related to image formed by a concave or convex mirror. In Section I of this paper, the assumptions made in arriving at these convenient rays for locating images formed by a concave mirror are discussed. In Section II, ray diagrams are sketched in a graph paper using the convenient rays for locating image formed by a concave mirror. Mirror formula is empirically verified by measuring the values of image distance, object distance and focal length of the concave mirror from the graph paper.

Section I

The reflection of light in case of images formed by a plane mirror occurs on plane surface. The *laws of reflection* are obeyed in plane mirror. In spherical mirror (here we will consider only concave mirror for simplification purpose) reflection of light occurs at curve surfaces. It must be remembered that the *laws of reflection* hold for curve surfaces also. At each point on the curved surfaces one can draw a surface tangent and draw the corresponding normal. Once we have a normal we can find the angle of incidence and the corresponding angle of reflection for a ray incident on it.

Let us look into how some of the convenient rays used for locating images formed by spherical mirrors are arrived at and what are the assumptions made to arrive at these rays. Draw a circle preferably with a dotted line, with point C as its centre as shown in Fig. 1. Consider a section MN marked on this circle as shown in the Fig. 1, so that this section represents a concave mirror. Let us mark the geometrical centre of the arc MN as pole P of the concave mirror. The line PC represents the principal axis of the concave mirror. Let us take a point X_1 on the surface of the concave mirror and draw a tangent at the point X_1 on the surface of the concave mirror. A line perpendicular to the tangent at the point X_1 will be normal to the curve surface at the point X_1 and will pass through the centre of the curvature C of the concave mirror. Since the point X_1 can be moved anywhere along the section of the concave mirror, by applying the geometrical properties of a sphere, a normal can be drawn at the corresponding point and we can always find the directions of the reflected ray by applying the *laws of reflection*.

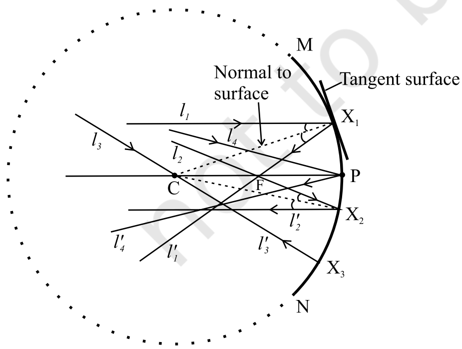

Fig.1: Reflection at a spherical surface

We can indentify more points say, X_2 and X_3 as shown in Fig. 1 and find the direction of the reflected ray. An incident ray l_1 parallel to the principal axis is incident at the point X_1, after reflecting from the concave mirror it is reflected as l'_1 and passes through the focus F. Let us apply the *laws of reflection* for the concave mirror at the point X_1, with the incident and reflected rays making the same angle θ_i and θ_r ($\theta_i = \theta_r$) with the normal to the surface at point X_1. Consider another incident ray l_2 passing through the focus F and incident at the point X_2, after reflection, the reflected ray l'_2 travels parallel to the principal axis. Similarly, we take incident rays l_3 and l_4, the reflected rays will travelled as l'_3 and l'_4 respectively as shown in the Fig. 1. Remember that all the rays considered above after reflection on the concave mirror obey *laws of reflection.*

Now coming to the image formation by a concave mirror, consider a point object A on the principal axis of the concave mirror as shown in Fig. 2. An infinite number of rays emanate from this point object. The rays then diverge from this point in all directions. The rays which get reflected from the concave mirror and converge at the image point I contribute to the image formation of the point object. As can be seen from Fig. 2, although all incident and reflected rays obey *laws of reflection*, only rays that strike the concave mirror near the axis AP, contribute to the image formation whereas rays that strike the mirror at points far from the principal axis on reflection pass near the image point and not on the image point.

Similarly, let us consider the rays incident on a concave mirror parallel to the principal axis as shown in the Fig. 3. All rays which are near the principal axis after reflecting from

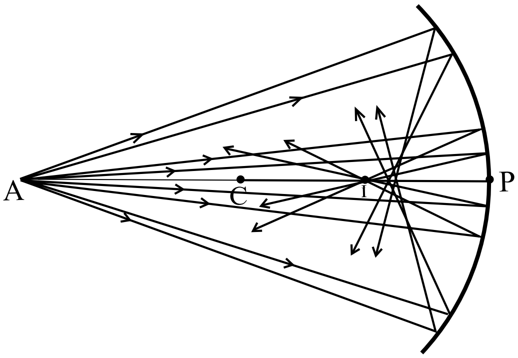

Fig. 2. Incident rays from a point source reflected by a concave mirror

the concave mirror pass through the focus F, whereas rays far off from the principal axis after reflecting from the concave mirror pass near the focus and not on the focus.

Remember that all the rays obey the law of reflection while reflecting from the concave mirror. Rays almost parallel with the principal axis and near to it are paraxial rays. Rays that strike the mirror at points far from

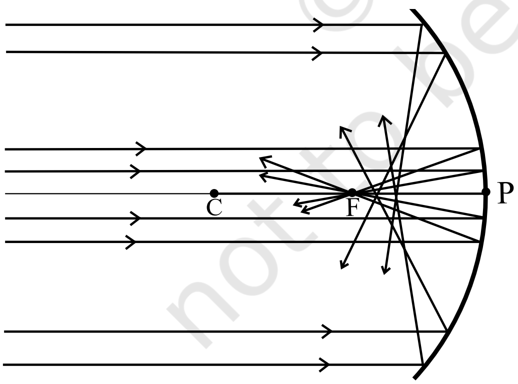

Fig. 3. Incident parallel rays reflected by a concave mirror

the principal axis upon reflection pass near the image point, but not through it. Such rays cause the image to blur and the effect is called spherical aberration. The image can be sharpened by blocking all non paraxial rays. To minimise spherical aberration a concave mirror having large focal length or large radius of curvature and small aperture is used. Thus the concave mirror which we will be referring is assumed that it has large focal length and small aperture. For such a concave mirror, $f = \frac{R}{2}$, where *f* is the focal length of the mirror and *R* is the radius of curvature of the mirror.

Thus the assumption we have made so far is that only paraxial rays contribute to the image formation and the mirror has a large focal length and small aperture. Although all paraxial rays reflected by the concave mirror contribute to image formation, some rays are more convenient to use due to the geometrical construct of spherical surfaces or symmetrical nature of the incident and reflected ray and not requiring us to measure every time the angle of incident and reflected rays. The convenient rays used for locating image formed by a concave mirror in most of the textbooks including Science Textbook, Class X, NCERT are listed below.

(i) A ray parallel to the principal axis, after reflection, will pass through the principal focus in case of a concave mirror [Fig.1].

(ii) A ray passing through the principal focus of a concave mirror after reflection will emerge parallel to the principal axis [Fig.1].

(iii) A ray passing through the centre of curvature of a concave mirror after reflection is reflected back along the same path [Fig.1].

(iv) A ray incident obliquely to the principal axis, towards a point P (pole of the mirror), on the concave mirror [Fig1.] is reflected obliquely. The incident and reflected rays follow the laws of *reflection* at the point of incidence (point P), making equal angles with the principal axis.

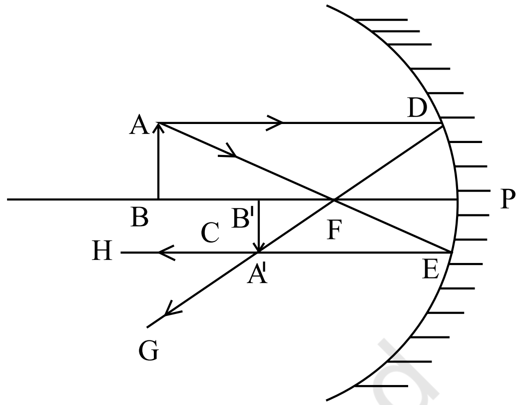

Fig. 4. Image formed by a concave mirror

Section II

Let us take a graph paper and draw a concave mirror of focal length, say 25 cm (or in grid unit) as shown in the Fig. 4. This may be taken as a representative Fig. and it is ensured that the mirror has a large radius of curvature and we are considering rays closed to the principal axis only.

We place an object AB on the principal axis at a distance u from the pole of the spherical mirror. Now, measure the distance of the object from the pole of the mirror (say in the unit of centimetre or in grid unit). Using a ray tracing technique we traced a ray AD parallel to the principal axis and after reflecting through the surface of the concave mirror it will pass through the principal focus F (Remember the ray which we have traced obey the *laws of reflection*) and travelled as ray DG as shown in Fig. 4. Similarly draw a ray AE which pass through the principal focus F and the ray after reflection on the surface of the concave mirror will travel parallel to the principal axis as EH as shown in the figure 4.

The ray DG and EH will meet at the point A'. This is the image of the object point A. Draw a line perpendicular to the ray EH from the point A' to meet at a point B' on the principal axis. Joined the point A'B' to obtain the image of the object AB. Now, measure the distance of the image from the pole to the point B'. This distance will give us the image distance v. From the ray we have traced on the graph paper we can verify the mirror formula. Substitute the value of the image distance, object distance and focal length in the mirror formula.

From the ray traced on the graph we can find the value of u, v and f in number of grid units.

Here, u= -12, v= -8 and f= -5 {Using Cartesian sign convention}

Putting these values in the mirror formula,

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$$

Let us put the value for left hand side,

$$\frac{1}{u}+\frac{1}{v}=-\left(\frac{1}{12}+\frac{1}{8}\right)=-\frac{5}{24}$$

Again let us put the value for right hand side,

$$\frac{1}{f}=-\frac{1}{5}$$

Thus left hand side value is very close to the right hand side value (Ideally this two values should be equal and if we take a more closely space grid line graph, we can more accurately verify the mirror formula).

Further, we can also find the magnification produced by the concave mirror.

$m=-\frac{v}{u}-\frac{8}{12}=-\frac{2}{3}$, the negative sign shows that the image formed is inverted.

We have discussed about the image formed by a concave mirror when an object is placed on the principal axis. Now let us consider an object placed slightly away from the principal axis as shown in the Fig. 5.

We place an object KL as shown in Fig. 5.

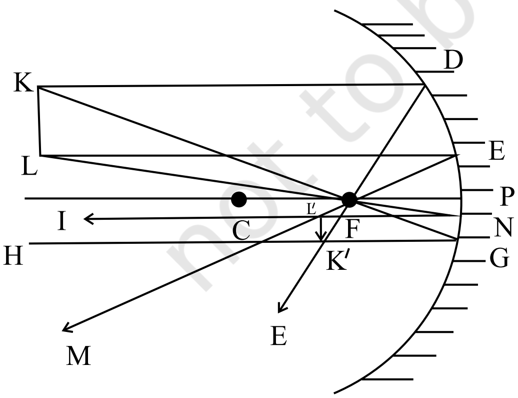

Fig. 5. Image formed by a concave mirror when an object is lightly placed away from the principal axis

Draw a ray KD parallel to the principal axis and after reflecting from the concave surface, it will pass through the principal focus F and travel as ray DE. We also traced a ray KG passing through the focus F and after reflection from the concave mirror, it will travel as ray GH parallel to the principal axis.

Similarly we can trace a ray LE parallel to the principal axis and after reflecting from the concave mirror, it will pass through the focus F as ray EM. Also we trace a ray LN passing through the focus and after reflecting from the concave mirror, it will travel as ray NI parallel to the principal axis. The ray DE and GH will meet at a point K' and the ray EM and NI will meet at the point L'. Now join the point K' and L'. The distance K'L' gives the size of the image.

Conclusion

The convenient rays used for locating images formed by concave mirror in ray tracing technique follow laws of reflection. Only paraxial rays reflected by the concave mirror contribute to image formation. Further, images formed by a concave mirror can be located using a ray tracing technique on a graph paper. Hence image formation by spherical mirrors can be taught through ray tracing on a graph paper and we can also empirically verify the mirror formula.

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