# PROBLEM-BASED LEARNING IN BASIC PHYSICS – VIII

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In this article, eighth in the series of articles, we present problems for a problem-based learning course based on dimensional analysis. We present the technique of dimensional analysis in the area of basic physics and what each problem tries to achieve with its solution.

## Introduction

The dimensions of physical quantity are the powers (or exponents) to which the base quantities are raised to represent that quantity (NCERT, 2006).

The applications of dimensional analysis are:

1. Checking the dimensional consistency of equations.
2. Deducing relations among physical quantities. The limitation of this is, we cannot relate number of quantities that may result in number of equations less than number of unknowns. Thus, number of quantities involved cannot be greater than the number of fundamental quantities whose dimensions are involved.
3. To find dimensions of a new quantity.

## Argument of a Function

**Most of the functions can be expanded into power series as shown below.**

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

$$e^x\ 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + -.........$$

In such cases, x has to be a dimensionless quantity otherwise we have right hand side of an equation in which each term has different dimension.

Now, consider famous equations like

1. $N = N_0 e^{-\lambda t}$ from radioactive decay
2. $q = Q\left(1 - e^{(-t/RC)}\right)$ for charging a capacitor in an RC circuit
3. $i = \frac{E}{R}\left(1 - e^{(-Rt/L)}\right)$ for growth of current in an LR circuit

   In all of the cases, quantities in the exponent have to be dimensionless and thus $\frac{1}{\gg}, \frac{1}{RC}, RC, and\ \frac{L}{R}$ and $\frac{L}{R}$ have dimensions of time and are called time constants. These quantities in some sense represent time scale at which changes occur in respective systems.
4. $y = A\sin(kx - \omega t)$ for one-dimensional simple

harmonic progressive wave has wavelength $\lambda = \frac{2\pi}{k}$ which has dimensions of length and time period of oscillation $T = \frac{2\grave{A}}{\acute{E}}$ which has dimensions of time.

1. Using dimensional analysis check the consistency of equation $T = 2\pi\sqrt{\frac{1}{g}}$ of time period of a simple pendulum on mass *m* of its bob, length *l* of its length and acceleration due to gravity *g*.
2. Excess (above atmospheric value) pressure inside a liquid is expected to depend on depth *h* below its surface, density *p* and gravity *g*. Using dimensional analysis, deduce relation between pressure P and *h*, *p*, *g*.
3. Using dimensional analysis establish— (i) dependence of potential energy of a particle of mass *m*, placed in the earth's gravitational field having acceleration due to gravity *g* at a height *h* above the earth's surface; (ii) dependence of kinetic energy of a particle of mass *m*, moving with velocity *v*; (iii) In this, it is not possible to determine constant of proportionality from dimensional analysis. However, using 2 and 3 above with the law of conservation of energy, and kinematical equations, determine the ratio of proportionality constants in 2 and 3.
4. Viscous force between two layers of liquid in motion is defined by Newton's formula as $F = \eta A \frac{dv}{dz}$ where *A* is the area of contact between the two layers, and $\frac{dv}{dz}$ is the velocity gradient in the liquid, i.e., velocity of the flow changes by *dv* in distance *dz* perpendicular to flow direction. Which are the dimensions of $\eta$, known as coefficient of viscosity? When a spherical object of radius R flows through a viscous medium with speed *v*, it experiences viscous drag (resistive force). Find an expression of this viscous force using dimensional analysis, assuming it to depend on $\eta$, R and *v*. In this case, it is not possible to determine constant of proportionality which turns out to be 6≠ and the formula thus obtained is known as Stokes' law.
5. A great physicist of previous century (P.A.M. Dirac) loved playing with numerical values of fundamental constants of nature. By playing dimensionally with mass of electron $m_e$ charge on electron *e*, Planck's constant *h*, and gravitational constant G we may be able to obtain expressions for what is known as Planck length *lP*, Planck time *tP* and Planck mass *mP*. Obtain these expressions dimensionally and estimate value of these quantities. Particle Physicists and Cosmologists use these values in trying to understand evolution of our universe.
6. From Coulomb's formula $\vec{E} = \frac{1}{4\pi \in_0}\frac{q}{r^2}\hat{r}$ and Bio-Savart's Law $\vec{B} = \int \frac{\mu_0}{4\pi \in_0}\frac{idl\, q}{r^2}\hat{r}$ find dimensions of $\varepsilon_0$ 0 and $\mu$0. Show that $\frac{1}{\sqrt{\varepsilon_0\mu_0}}$ has dimensions of velocity. From the values of $\varepsilon_0 = 8.85\times10^{-12}$ SI units and $\mu_0 = 4\pi\times10^{-7}$ SI units, estimate value of this velocity.
7. A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When electrons are subjected to electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions

with a natural angular frequency *'Ep'* , which is called plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency $\omega$ , where a part of energy is absorbed and a part of it is reflected. As $\omega$ approaches $\sqrt{\frac{Ne}{m\varepsilon_0}}$ all the free electrons are set to resonate together and all the energy is reflected. This is explanation of highly reflective metals.

(i) Taking the electronic charge as *'e'* and the permittivity as 'εo', use dimensional analysis to determine the correct expression for ω p.

(ii) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (b) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (c) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (d) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

(iii) Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N = 4 \times 10^{27} m^{-3}$. Take $\in_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units.
(a) 800nm (b) 600nm (c) 300nm (d) 200nm

[JEE, 2011]*

## Hints, Solutions, Answers:

1. $[l] = [L]^1 \quad [g] = [L]^1 [T]^{-2}$. Thus

$$\left[\sqrt{\frac{l}{g}}\right] = \left[\frac{[L]^1}{[L]^1[T]^{-2}}\right]^{1/2} = [T]$$

The given equation is dimensionally correct.

2. Let $\alpha\, h^x p^y g^z$ here
$[p] = [F/A] = [M^1 L^1 T^{-2}]/[M]^1 [L]^{-1} [T]^{-2}$
$[h] = [L]^1, [A] = [M]^1 [L]^{-3}$ and $[g] = [L]^1 [T]^{-2}$
Taking $[p] = [h]^x [A]^y [g]^z$, and equating dimensions of m, L and T, we get
(power of [M]) : *y=1,*
(power of [L]) : $x-3y+z=-1$
(power of [T]) : $-2z=-2$
This gives us z = 1 and x = 1 and we have
$p\, \alpha\, hpg$

3. We get $PE\alpha$ mgh and 2 $KE\alpha mv^2$ using conservation of energy and kinematical equation. If we choose system of units such that $PE = mg$ then $KE = \frac{1}{2}mv^2$

4. $F \alpha \eta^x R^y v^z$ and $[F] = [\eta]^x [R]^y [v]^z$
$\therefore [M^1 L^1 T^{-2}] = [M^1 L^{-1} T^{-1}]^x [L]^y [L^1 T^{-1}]^z$
Matching the powers on two sides, this gives us *x* = 1, *y* = 1 and *z* = 1.
Thus , F αη R v , the constant of proportionality 6≠ is determined from some other method.

5. We get $t_p - \left(\frac{Gh}{c^5}\right)^{1/2} = 1.35 \times 10^{-43}$ sec
$l_r \alpha \left(\frac{Gh}{c^5}\right)^{1/2} = 4.05 \times 10^{-35}$ and
$m_p \alpha \left(\frac{ch}{G}\right)^{1/2} = 5.46 \times 10^{-8}$ kg

6. Taking
$[q] = [I^1 T^1], [E] = [F]/[q]$ and $[B] = [F]/[q][v]$ and $[B] = [F]/[q][v]$, we get
$[E] = [M^1 L^1 T^{-3} I^{-1}]$ and $[B] = [M^1 T^{-2} I^{-1}]$
This gives us $[\varepsilon_0] = [M^{-1} L^{-3} T^4 I^2]$
and $[\mu_0] = [M^1 L^1 T^{-2} I^{-2}]$
Thus, $\left[\frac{1}{\sqrt{\varepsilon_0 \mu_0}}\right] = [L^1 T^{-1}]$ and from the given values $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8$

7. [Ans: (i) - (c) , (ii) - (b)]

*JEE–Joint Entrance Examination for admission to IIT.

# Reference

National Council of Educational Research and Training. 2006. *Physics Textbook for Class XI, Part-I.* New Delhi.

JEE - Joint Entrance Examination for admission to IIT 2011.