# PROBLEM BASED LEARNING IN BASIC PHYSICS - VIII

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## Introduction

In this article, seventh in the series of articles, we present problems for a problem based learning course from the area of material properties. We present the learning objectives in this area of basic physics and what each problem tries to achieve with its solution.

Methodology and philosophy of selecting these problems have already been discussed. (Pradhan 2009, Mody 2011)

To review methodology in brief, we note here that this PBL (problem based learning) starts after students have been introduced to formal structure of physics. Ideally, students would attempt only the main problem. If they find it difficult, then depending upon their area of difficulty, right auxiliary problem have to be introduced by a teacher who is expected to be a constructivist facilitator. The teacher may choose as per his/her requirement or may construct questions on the spot to guide student to right idea and method.

## Material Properties: Elasticity of Solids and Fluids, as well as, Mechanics of Fluids

**1.** To understand the role that property of elasticity of solid, liquid and gases plays within material according to Hooke's law along with laws of physics and how it works in known situations.

**2.** To understand the behaviour of fluids according to Archimedes principle, equation of continuity and Bernoulli's principle.

## Problems

**1.** A sphere of mass 1 kg is suspended at the end of 2 m long steel wire whose other end is fixed to the ceiling. The wire has a cross-sectional area of 1 mm$^2$. The sphere is raised by some height and then dropped to give sudden jerk to the wire. Find the maximum height to which the ball can be raised so that wire doesn't break. [Given: Y-steel= 2×1011 N/m$^2$ and breaking stress for steel = 1.1×109 N/m$^2$]

**Tasks involved in this problem are:**

a. to relate spring constant with elastic modulus.

b. to apply conservation of energy to the wire to find its extension assuming the model of elastic spring.

c. to calculate stress based on extension of the wire.

d. to estimate maximum height that will not allow stress to exceed is breaking limit.

**2.** A rod of length 1.05 m having negligible mass is supported at its ends by two wires, A of steel and B of aluminium of equal lengths as shown in figure. The cross-section areas of wires A and B are 1.0 $mm^2$ and 2.0 $mm^2$ respectively. At what point along the rod should a mass m be suspended in order to produce (a) equal stress and (b) equal strains in both the wires?

[Given: Y steel = $20\times10^{10}$ N/$m^2$Yaluminium = $7\times10^{10}$ N/$m^2$] [NCERT XI]

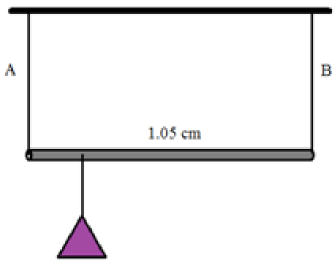


Fig. 1

**Tasks involved in this problem are:**

a. to apply condition of equilibrium under given situation.

b. to calculate position where mass to be hanged to meet the required condition.

**3.** A wire of density 9 gm/$cm^3$ is stretched between two clamps 100 cm apart, while subjected to an extension of 0.05 cm. What is the lowest frequency of transverse vibrations in the wire, assuming Young's modulus of the material to be $9\times10^{11}$ dyne/$cm^2$? [JEE 1975]

**Tasks involved in this problem are:**

a. to calculate tension due to stretching of wire.

b. to calculate fundamental frequency of vibration based on this tension.

**4.** A rail track made of steel having length 10 m is clamped on a railway line at its two ends. On a summer day due to rise in temperature by 20°C, it is deformed as shown in the figure. Find x, (displacement of the centre) if $\alpha$ steel = $1.2\times10^{-5}$ /°C andYsteel = $20\times10^{10}$ N/$m^2$] [NCERT EP XI]

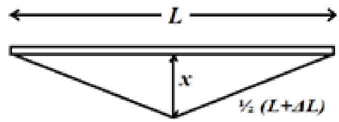


Fig. 2

**Tasks involved in this problem are:**

a. To calculate increase in length in accordance with property of thermal expansion.

b. To find geometric deformation.

**Fluids**

**5.** A piece of brass (alloy of copper and zinc) weighs 12.9 g in air. When completely immersed in water it weighs 11.3 g. What is the mass of copper contained in the alloy? Specific gravity of copper and zinc, are 8.9 and 7.1 respectively. [JEE 1966] [Note: This is the problem similar to what Archimedes must have faced when he was assigned the task to estimate gold in the crown.]

**Tasks involved in this problem are:**

a. to frame the equations for total mass and loss of weight.

b. to realize that it is volume of the body that decides the weight of the liquid displaced and hence loss of weight.

c. solve the two equations for volume of one of the constituent and hence estimate the masses.

[Here auxiliary problems are given to assist in realizing formation of simple equation for loss of mass and how it relates to volume of the object.]

**Auxilliary Problems**

a. What percentage of volume of ice remains submerged while it is floating on the water surface? Ice has specific gravity is 0.91.

b. A piece of copper having an internal cavity weighs 264 g in air and 221 g in water. Find the volume of the cavity. Density of copper = 8.8 g $cm^{-3}$. [JEE 1963]

**6.** A boat carrying number of large stones is floating in a water tank that is about to overflow. If the stones are unloaded into the water tank, what will happen to water level in water tank? Will water in the tank over flow? Give scientific explanation based on Archimedes principle.

**Tasks involved in this problem are:**

This is not necessarily a mathematical problem. However one can write conditions to arrive at the conclusion. One can also argue qualitatively.

**7.** A cylindrical vessel of height 500 mm has an orifice (small hole) at its bottom. The orifice is initially closed and water is filled in it up to height H. Now the top is completely sealed with a cap and the orifice at the bottom is opened. Some water comes out from the orifice and the water level in the vessel becomes steady with height of water column being 200 mm. Find the fall in the height (in mm) of water level due to opening of the orifice. [Take atmospheric pressure = $1.0 \times 10^5$ N/$m^2$, density of water = 1000 kg/$m^3$ and g = 10 m/$s^2$. Neglect any effect of surface tension.] [JEE 2009]

**Tasks involved in this problem are:**

a. to realise and write equation for the condition that has to be satisfied once the orifice is opened for liquid to stop flowing out.

b. to understand the thermodynamic condition of air trapped above the water.

c. to solve the two conditions to estimate the loss of height of water.

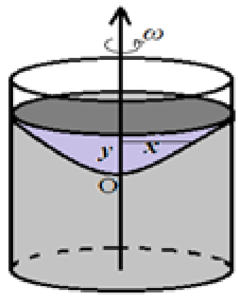


**Fig. 3**

**8.** Water in a cylinder is being rotated at constant angular speed ω about the axis of the cylinder (see the figure). Ignore effect of surface tension and find the equation of the surface of water. Can you identify the surface?

**Tasks involved in this problem are:**

a. to find the net force direction on a mass element on the liquid surface so that surface assumes position perpendicular to the force.

b. geometrically finding the angle to arrive at equation of the surface.

**9.** A non-viscous liquid of constant density 1000 kg/m3 flows in a streamline motion along a tube of variable cross-section. The tube is kept inclined in the vertical plane as shown in the figure 3. The area of cross-section of the tube at two points P and Q at heights of 2 metre and 5 metre are respectively 4×10 – $m^2$ and 8×10 – 3$m^2$. The velocity of the liquid at point P is 1 m/s. Find the work done per unit volume by the pressure and the gravity forces as the fluid flows from point P to Q. [JEE 1997]

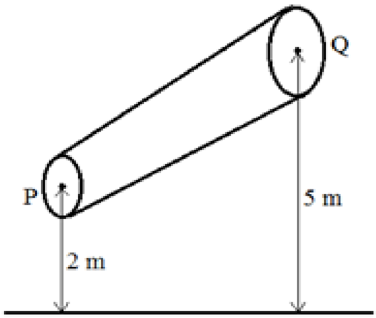


Fig. 4

**Tasks involved in this problem are:**

a. to apply equation of continuity to estimate speed of the fluid at the other end.

b. to calculate pressure difference at the two ends using Bernoulli's principle.

c. to estimate the work/energy accordingly.

**10.** A uniform wire having mass per unit length $\lambda$ is placed over a liquid surface. The wire causes the liquid to depress by y ($y < a$) as shown in the figure. Find the surface tension of the liquid. Neglect end effect. [JEE 2004]

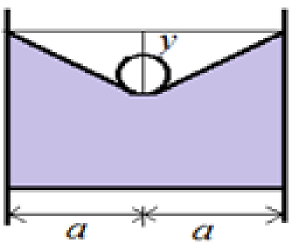


Fig. 5

**Tasks involved in this problem are:**

a. to understand using free body diagram as how does the force of surface tension support this object to float on the liquid surface.

b. to understand how and why small angle approximation have to be applied to solve the equations obtained in part 'a' to get the expression for surface tension.

**11.** A soap bubble having surface tension T and radius R is formed on a ring of radius b ($b < R$). Air is blown inside tube with velocity v as shown. The air molecules collide perpendicularly with the wall of the bubble and stops. Calculate the radius at which the bubble separates from the ring.
[JEE 2003]

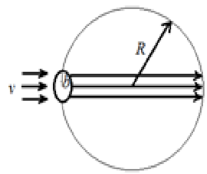


Fig. 6

**Tasks involved in this problem are:**

a. to apply Bernoulli's principle inside and outside the bubble.

b. to use excess pressure inside the bubble concept to understand why bubble would separate away from the ring. [Remember this is a popular toy-based problem which children play with.]

**12.** Eight droplets of mercury, each of radius 1 mm, coalesce into a single drop. Find (i) the radius of the single drop formed (assuming all the droplets and the drop to be spherical in shape), (ii) the change in the surface energy of the mercury drop, (iii) the change in the temperature of the mercury. [Surface tension of mercury = 0.465 J/m$^2$, Density of mercury = 13.6×10$^3$ kg/m$^3$, Specific heat of mercury = 140 J Kg$^{-1}$ K$^{-1}$]

[Ans: (i) 2 mm (ii) 2.337×10$^{-5}$ J (iii) 3.663×10$^{-4}$ K]

**Tasks involved in this problem are:**

a. to understand that volume /mass of the liquid drop/s remain same when they combine or breakup.

b. to understand need for energy in breaking or release of energy when drops combine.

c. to realise what happens to excess (surface) energy in accordance with energy conservation principle.

## Solutions

**1.** Let the sphere be raised to height h. When it reaches the bottom, the gravitational P.E. at the bottom point gets converted in to elastic P.E. due to extension of the wire.

i.e., mgh = ½ $kx^2$ where x is the extension of the wire. [Here $k = \frac{YA}{L}$ from Solution of P(2) above.]

This strain causes $\text{Stress} = \frac{Y}{L}x = \frac{Y}{L}\left(\frac{2mghL}{YA}\right)^{1/2}$

To prevent breaking this has to be less than breaking stress SB. Which gives the condition

$$h < \frac{ALS_B^2}{2mgY}$$

**2.** Let $T_A$ and $T_B$ represent tensions in string A (steel) and B (aluminum) respectively, then

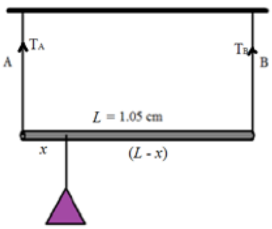


**Fig. 7**

(a) For equal stress : $\frac{T_A}{A_A} = \frac{T_B}{A_B}$ and for torque around point of suspension of weight to be in equilibrium, $T_A x = T_B (L - x)$

(b) For equal strain : $\Delta l_A = \Delta l_B$ which gives $\frac{T_A Y_A}{A_A} = \frac{T_B Y_B}{A_B}$ and for torque around point of suspension of weight to be in equilibrium, $T_A x = T_B (L - x)$-

Solving the two equations gives :

$$x = \left(\frac{A_B Y_A}{A_B Y_A + A_A Y_B}\right)L = \frac{7}{47}\times 1.05 = 0.1564\text{m}$$

**3.** For a wire extended under tension,

$T = Y\frac{\Delta l}{L}A$ and frequency of vibrating string

$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ where m is mass per unit length of the wire.

In terms of given quantities, m = ρA which gives

$$n = \frac{1}{2l}\sqrt{\frac{Y\Delta l}{\rho l}}$$

**4.** Referring to the figure given and applying Pythagoras,

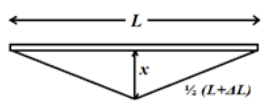


**Fig. 8**

$x^2 = \frac{1}{4}(L^2 + 2L\Delta L) - \frac{1}{4}L^2$ (neglecting higher order term of $\Delta L$ we get $x^2 = ½ L\Delta L$,

Which gives $x = \sqrt{\frac{L\Delta L}{2}} = L\sqrt{\frac{\alpha T}{2}} \approx 11cm$

**5.** Let 1 denote Copper and 2 denote zinc.

The given information than gives $m^1 + m^2 = p_1V_1 + p_2V_2 = 12.9$ gm and $pw(V_1 + V_2) = 1.6$ gm

Substituting appropriate specific gravities (note $pw$= 1) , we get $m_1 = 7.61$ gm

## Auxiliary Solutions

A. For the floating ice, let *V* be the volume of ice and *V'* be the volume of the submerged portion.

Weight of the ice = weight of the water displaced

∴ *PiceVg* = *P* water*V'* g

$$\therefore \frac{V'}{V} = \frac{V_{water}}{V_{ice}} = \frac{\rho_{ice}}{\rho_{water}} = 0.91$$

B. Let V be the volume of the metal piece and V0 be that of cavity.

Then we have $p(V - V_0) = 264$ and $pwV = 264 - 221 = 43$

Solving which we get $V_0 = 13\ cm_3$.

**6.** Volume of water displaced = (weight of the stone)/($p$waterg)

and volume of stone = $p$stone *V*stoneg

this gives $V_{water} = \left(\frac{\rho_{stone}}{\rho_{water}}\right)V_{stone}$ and since

*P*water<*P*stone we get *V*stone<*V*water

The tank will not over flow.

**7.** Initially the pressure of air above the liquid is $P_A$ : the atmospheric pressure after the orifice is opened, the pressure of air above the liquid is $P = P_A - hpg$ and also $P_A\ (L - H)\ A = P\ (L - h)$ A where A is the area of cross-section of the cylinder and *L* =*500* mm and *h* = *200* mm. findingP and solving the equations we get H = 206 mm so that *H - h = 6 mm.*

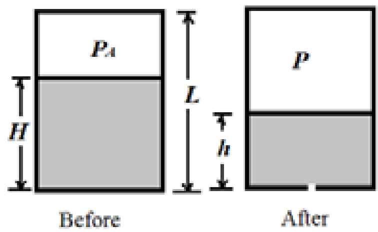


Fig. 9

**8.** Referring to figure above, $tan\,\theta = \frac{dy}{dx} = \frac{m\omega^2 x}{mg}$ for a small element of mass mat *(x, y)*

Integration yields, $y = \left(\frac{\omega^2}{2g}\right)x^2$ , which is equation of parabola.

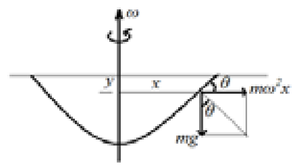


Fig. 10

**9.** From equation of continuity $v_pA_p = v_qA_q$ that gives vq = ½ m/s.

Work done by gravity = $pgh = 29.4{\times}10^3\ J/m^3$

From Bernoulli's equation we have $P_p + ½\, pvp^2 = Pq + ½\, pvq^2 + pgh$

*This gives work by pressure force =* $P_p - P_q = pgh - (½\, pvp^2 - ½\, pvq^2) = 29.03{\times}10^3\ J/m^3$

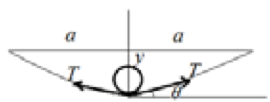


Fig. 11

**10.** Referring to the figure, when the needle is in equilibrium, $\tan\theta = \frac{y}{a}$ and $2T\sin\theta = \lambda g$ in small angle approximation: $\tan\theta \approx \sin\theta$ wihich gives

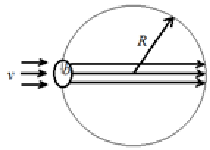


**Fig. 12**

**11.** Referring to figure : and using Bernoulli's equation for outside the bubble and inside the bubble, we get Pout + ½ ρvout$^2$ = Pin + ½ ρvin$^2$

$P_{in} = \frac{4T}{R} + P_{out}$ where *T* is the surface tension and $v_{in} = 0$ yields $R \geq \frac{8T}{\rho v^2}$

**12.** When drops coalesce into one, the total volume remains constant : i.e.,

$8 \times \frac{4\pi}{3} r^3 = \frac{4\pi}{3} R^3$ which gives *R = 2r = 2 mm*

Also loss of surface energy results in rise in temperature of the drop.

i.e., $\Delta U_{surface} = ms\Delta\theta$

$$\Delta U = T\left(8(4\pi r^2) - 4\pi R^2\right) = 2.337 \times 10^{-5}\,J$$

which gives $\Delta\theta = 3.663 \times 10^{-4}$ K

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