PROJECTILE MOTION OF A CRICKET BALL FROM BOWLING TO OVER BOUNDARY IN CRICKET

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In a cricket game, bowler bowls at the batsman who hits the ball powerfully and hits an over boundary. In this process, first considered projectile motion of the ball from the bowler to the pitch where the ball strikes affecting an elastic impact with the grounds followed by its second projectile motion until being batted by the batsman causing the second impact but between the ball and the bat. The third projectile motion arises from the batsman to a spot outside the field resulting in score of an over boundary, So that in course of projectile motion of the cricket ball vis-a-vis its impact with the pitch and the bat a minimum velocity of hitting the ball with the bat by the batsman for over boundary is determined. Thereafter the maximum range in the horizontal direction of the ball due to a given velocity of hitting it, escaping a catch by a fieldsman, is also computed. All along the air resistance is neglected. The effects of spinning of the ball are neglected.

Introduction

Khillare (2006) dealt with the projectile motion of a cricket ball bowled by a player upto the pitch or upto the batsman in case of full-toss ball and thereafter with direct/oblique impact between the ball and the bat. In order to determine the velocity of the ball immediately after its impact with the bat, he considers that this velocity is in the direction normal to the bat surface, whereas from a realistic point of view, the batsman has all options to strike the ball in different directions which tends to affect the velocity of the ball. In his paper, Khillare (2006) has shown that the bowler throws the ball upward making an angle with the horizon to determine its range upto the pitch or batsman, which is far from a realistic situation. In fact the bowler throws the ball downwards making an angle with the horizon aiming at its reaching the pitch/batsman in so much as this leads to far greater velocity of approach of the ball towards the batsman who is then exposed to greater chances of misplaying/commiting mistakes causing bowled-out, caught-out or L.B.W. In Khillare's treatment, the cricket ball bowled by a player attains the greatest height and afterwards goes down to reach the pitch/batsman in a longer time. In contrast while watching a cricket match, the bowler is mostly observed to throw the ball downwards at a certain angle — horizon, i.e., not allowing it to attain the maximum height but for it to reach to the pitch or batsman in faster time. In the second phase, he considers the elastic impact of the ball with the pitch, taking into account the coefficient of elasticity between the ball and the ground. In the third, fourth and fifth phases the motion of the ball from the pitch to the batsman at a certain range, elastic collision between the bat and the ball, and motion of the ball lifted by the batsman in air, ultimately scoring an overboundary have been discussed.

Motion of Cricket Ball from Bowler to the Pitch

Let the bowler deliver the ball from height h_1 measured from the ground downwards with velocity "u" an angle "α" to the horizontal and pitch "t" at a distance x_1 from himself in time t_1, g being the acceleration due to gravity neglecting the air resistance and considering its motion in the vertical and horizontal direction, as illustrated in Fig. 1.

$$h_1 = u\sin\alpha.t_1 + \frac{1}{2}gt_1^2 \qquad (1)$$

$$x_1 = u\cos\alpha.t_1 \qquad (2)$$

Had the ball been thrown upwards by the bowler at that angle keeping all other parameters the same, the ball would reach pitch in time t_1' covering the horizontal distance .

$$-h_1 = u\sin\alpha.t'_1 - \frac{1}{2}gt_1^2 \qquad (3)$$

$$x'_1 = u\cos\alpha.t_1' \qquad (4)$$

Comparison between equations (1) and (3) corroborates that $t_1' > t_1$ and consequently with a chance of yielding a full toss ball.

Eliminating t_1 between (1) and (2) one gets

$$h_1 = x_1\tan\alpha + \frac{1}{2}g\frac{x_1^2}{u^2}\sec^2\alpha \qquad (5)$$

Bounce of the Ball from the Pitch

Let the ball strike the pitch with velocity v_1 at angle to the level ground. Then according to textbook' the relation between the initial and final velocities in the vertical direction is given by

$$v_1^2\sin^2\beta = u^2\sin^2\alpha + 2gh_1 \qquad (6)$$

While the horizontal component of the velocity remain constant

$$v_1\cos\beta = u\cos\alpha \qquad (7)$$

Considering an elastic collision[2] of the ball with the pitch having the coefficient e_1 of elasticity between them, by Newton's experimental law of collision, one can write

$$v_2\sin\gamma = e_1 v_1\sin\beta \qquad (8)$$

Where v_2 is the velocity of the ball after rebound the pitch at an angle γ to the horizontal. This is illustrated also in Fig. 1. After all the horizontal component of the ball velocity remaining constant because of no gravitational force and no impulsive action in that direction, we can rewrite

$$u\cos\alpha = v_1\cos\beta = v_2\cos\gamma \qquad (9)$$

Thereafter if the cricket ball attain a height h_2 from the pitch (to be stricker by the batsman) and thus travels a horizontal distance x_2 along the pitch in time t_2, then

$$h_2 = v_2\sin\gamma t_2 - \frac{1}{2}g{t_2}^2 \qquad (10)$$

$$x_2 = v_2\cos\gamma t_2$$

Or, because of (9), it becomes

$$x_2 = u\cos\alpha.t_2 \qquad (11)$$

Eliminating t_2 between (10) and (11) one gets

$$h_2 = \frac{x_2 v_2 \sin\gamma}{u\cos\alpha} - \frac{1}{2} g \frac{x_2{}^2}{u^2} \sec^2\alpha \qquad (12)$$

Further by use of (6) and (8), one gets for (12) :

$$h_2 = \frac{x_2 e_1 \sqrt{u^2 \sin^2\alpha + 2gh_1}}{u\cos\alpha} - \frac{1}{2} g \frac{x_2{}^2}{u^2} \sec^2\alpha \qquad (13)$$

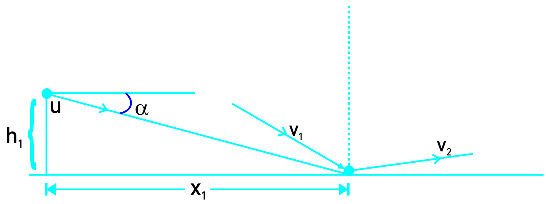

Fig. 1: The Ball Delivered by the Bowler is Bounced from the Pitch and Approaches the Batsman.

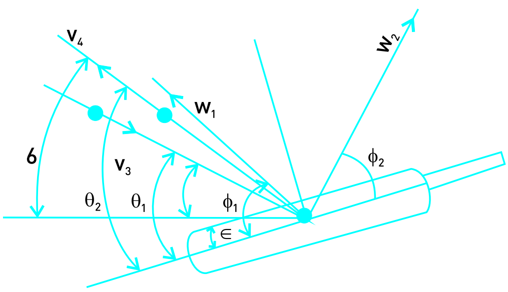

Fig. 2: The incoming Ball is Batted and Levels the Bat.

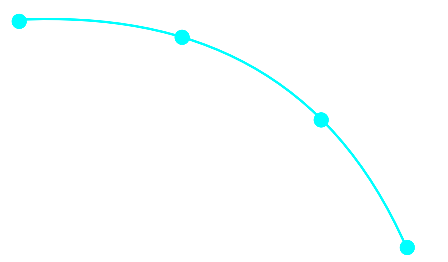

Fig. 3: The Ball Smashed by the Batsman Soars into the air Resulting in a Six.

If v_3 be the velocity of the ball and δ the angle of its inclination to the horizon after attaining the height h_2 from the pitch,

$$v_3{}^2 \sin\delta = v_2{}^2 \sin^2\gamma - 2gh2 \qquad (14)$$

$$v_3 \cos\delta = v_2 \cos\gamma = v_1 \cos\beta = u\cos\alpha \qquad (15)$$

Employing (8) in (14), one gets

$$v_3{}^2 \sin^2\delta = e_1{}^2 (u^2 \sin^2\alpha + 2gh_1) - 2gh_2 \qquad (16)$$

Impact Between the Bat and the Ball

Let the batsman strike the ball moving with velocity v_3 inclined at angle δ to the horizontal as above obviously resulting in an oblique elastic impact between the bat and the ball such that just before the impact the velocity of the bat is w_1 making and angle ϕ_1 to the plane surface of the bat at the point of contact between the bat and the ball is angle $(90-\phi_1)$ to the normal to the surface at the point of contact and the velocity of the approaching ball is v_3 whose direction makes an angle θ_1 in the same way with the plane surface of the bat, entailing

$$\theta_1 = \in + \delta \qquad (16\text{-}1)$$

Where $\in$ is the inclination of the bat surface with the horizontal.

So much so that let v_4 be the velocity of the ball immediately after the impact between the ball and the bat i.e. the velocity with which the ball leaves the bat, say, at an angle θ_2 to the bat surface and w_2 the velocity of the bat inclined at an angle ϕ_2 to the bat surface at the point of contract after the impact owing to the batsman's striking the ball with the bat. This is depicted in Fig. 2. Since the components of velocities perpendicular to the line of impact, i.e., along the line of intersection between the plane surface of the bat, and the plane containing the bat—normal at the point of contact of the bat with the ball at the instant of strike by the batsman and the direction of approach velocity of the ball before the impact remain unaltered before and after the impact because of no impulsive action along that line, we get

$$v_4 \cos\theta_2 = v_3 \cos\theta_1 \tag{17}$$

$$w_1 \cos\phi_1 = w_2 \cos\phi_2 \tag{18}$$

By the principle of conservation of momentum (Loney, 1969) along the line of impact i.e. normal to the surface of the bat of mass M striking the ball of mass m, one can write

$$mv_4 \sin\theta_2 + Mw_2 \sin\phi_2 = -mv_3 \sin\theta_1 + Mw_1 \sin\phi_1 \tag{19}$$

$$v_4 \sin\theta_2 - w_2 \sin\phi_2 = -e_2(-v_3 \sin\theta_1 - w_1 \sin\phi_1) \tag{20}$$

Which is due to Newtons experimental law (Loney, 1969) of collision, e_2 being the coefficient of elasticity between the ball and the bat. Combining (19) and (20),

$$V_4 \sin\theta_2 = \frac{M(1+e_2)w_1 \sin\theta_1 - (m - Me_2)v_3 \sin\theta_1}{M+m} \tag{21}$$

$$w_2 \sin\phi_2 = \frac{(M - e_2 m)w_1 \sin\phi_1 - m(1+e_2)v_3 \sin\theta_1}{M+m} \tag{22}$$

Dividing (21) by (17), we obtain the direction of the velocity of the ball heading towards over boundary which of course solely depend upon the batsman's prowess and strength of hitting the ball corroborated by the v_4 and θ_2 as above, escaping a fieldsman's catch and passing over the boundary line. However, squaring and adding (21) and (17) is obtained the velocity v_4 acquired by the ball after being striken by the batsman in an attempt to credit an over boundary.

$$V_4 = \left[\left\{\frac{M(1+e_2)w_1 \sin\theta_1 - (m - Me_2)v_3 \sin\theta_1}{M+m}\right\}^2 + v_3^2 \cos^2\theta_1\right]^{1/2} \tag{23}$$

But the ball leaves the bat for an overboundary at an angle σ to the horizon given by

$$\sigma = \theta_2 - E \tag{24}$$

Conditions for over Boundary

If the ball, after being hit by the batsman as above, strikes the ground beyond the boundary line covering a horizontal range R from him in time t_3 then

$$-h_2 = v_4(\sin\sigma)t_3 - \frac{1}{2}gt_3^2 \tag{25}$$

$$R = v_4(\cos\sigma)t_3 > R_0 \tag{26}$$

$$t_3 = \left[v_4 \sin\sigma + \sqrt{v_4^2 \sin^2\sigma + 2gh}\right]/g \tag{27}$$

Where R_0 is the distance of the boundary line from the batsman smashing the ball for an over-boundary. Relation (23) suggests that the more is the speed of the bat hitting the ball, the more is the velocity of the ball leaving the bat, however this velocity of the ball also depends on the angle ϕ_1 of posing the bat and contributes to its lifting in the air for safely crossing the boundary evading any scope of "catch" by any fieldsman.

Maximum Range of the Cricket Ball with given Velocity of the Ball Leaving the Bat

Now we can find the maximum range of the ball proceeding for an over boundary with respect to an optimum inclination of the velocity of ball acquired immediately after being hit by the batsman. This can be obtained by eliminating t_3 between (26) and (27) and thereafter applying differential calculus for maxima/minima.

$$-h_2 = R\tan\sigma - \frac{1}{2}gR^2(\sec^2\sigma)/v_4^2 \tag{28}$$

Differentiating (28) with respect to and setting

$$\frac{dR}{d\sigma} = 0 \tag{29}$$

for maxima/minima of R, one gets

$$R_{\max} = \frac{v_4^2}{g\tan\sigma_{opt}} \tag{30}$$

Eliminating R_{max} between (28) and (30), we get

$$-h_2 = \frac{v_4^2}{g} - \frac{v_4^2}{2g}\frac{(1+\tan^2\sigma)}{\tan^2\sigma}$$

Or, $$\tan^2\sigma = \frac{2gh_2}{v_4^2} + 1 \qquad (31)$$

Employing which in (30) we find the maximum range

$$R_{max} = \frac{v_4^2}{g\sqrt{1+\frac{2gh_2}{v_4^2}}} \qquad (32)$$

It can also be shown that $\left(\frac{d^2R}{d\sigma^2}\right)_{\sigma opt} < 0$ as the condition for maximum range.

The minimum velocity with which the ball can be relieved of the bat due to strike by the batsman for a given horizontal range R can be obtained by use of $\frac{d(v_4^2)}{d\sigma} = o$ from (28):

$$\left(v_4^2\right)_{min} = g\left(\sqrt{h_2^2 + R^2} - h_2\right) \qquad (33)$$

$$\left(\tan\sigma_{opt}\right) = \left[-h_2 + \sqrt{h_2^2 + R^2}\right]/R \qquad (34)$$

σ_{Opt} is the optimum angle of projection of the just batted ball to obtain the maximum range R_{max} with given initial velocity v_4 or the minimum velocity $(V_4)_{min}$ with given range R.

Numerical Example

In this section it is discussed with the help of a table how the batsman elegantly hits the ball for overboundaries. The cricket ground is of oval, i.e., elliptic shape with approximate distance 72 metre to 82 metre between the boundary line and the pitch, i.e., position of the batsman. So employing (27) and (32) is prepared Table 1 with given data mostly effecting a six. The average velocity of just batted ball varies from 90km/hr to 120km/hr for a sixer in a cricket match. The acceleration due to gravity = g= 10 metre/sec^2

Table 1

Velocity of the batted ball (metre/ sec.)	Exit Velocity of the batted ball (km/hr)	Inclination of the exit velocity with the horizontal (degree)	Height of the ball in contact with the bat (metre)	Time of the flight of the ball on striking the ground (second)	Horizontal distance covered by the ball (metre)	Distance of the boundary line from the batsman in the plane of the flight of the ball (metre)	The greatest height attained by the ball (metre)	Over boundary Yes or No
30	108	60°	1.00	5.22	78.165	75.12	34.75	Yes
28	1008	30°	1.00	2.87	69.616	72.52	10.80	No
30	108	30°	1.20	3.08	80.018	78.14	34.95	Yes
25	90	45°	1.50	3.621	64.010	70.15	17.125	No
30	108	45°	1.25	4.303	91.278	80.25	23.75	Yes
27	972	45°	1.25	4.025	76.845	74.35	19.475	Yes
28	100.8	60°	1.00	4.895	68.430	81.15	30.40	No
30	108	Sin-3/5	1.00	3.655	87.720	80.38	17.20	Yes
30	108	Sin-14/5	1.00	4.842	87.156	79.52	29.80	Yes
27	972	60°	1.00	4.719	63.714	73.25	28.34	No

Table 5.1 spells out that a batsman can strike a six by lifting the batted ball high after imparting it higher velocity. Nevertheless the batsman can drive the ball with comparatively lesser high speed followed by attaining lesser height in lesser time to get it over boundary. Otherwise the batted ball falls short of the boundary line to be caught by a fieldsman or rolls on the ground to the boundary line for a four.

References

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