PROBLEM-BASED LEARNING IN BASIC PHYSICS - V

A. K. Mody
V. E. S. College of Arts, Science and Commerce
Sindhi Society Chembur, Mumbai

H. C. Pradhan
HBCSE, TIFR, V. N. Purav Marg,
Mankhurd, Mumbai – 400 088

In this article, fifth in the series, we present problems for a problem-based learning course in the area of electricity and magnetism. We present the learning objectives in this area of basic physics and what each problem tries to achieve through its solution.

In this article, fifth in the series, we present problems on electricity and magnetism. Methodology and philosophy of selecting these problems have already been discussed (Mody and Pradhan 2011).

To review the methodology in brief, we note here that this Problem-based Learning (PBL) starts after students have been introduced to formal structure of physics. Ideally students would attempt only main problem. If they find it difficult, then depending upon their area of difficulty, right auxiliary problem have to be introduced by teacher who is expected to be a constructivist facilitator. Teacher may choose as per her/his requirement or may construct questions on the spot to guide student to the right idea and method.

Problems on Electricity and Magnetism

Learning Objectives

1. Coulomb's law: electric force, electric field and electric potential.
2. The fact that force and field are vectors whereas potential is a scalar and how they are to be calculated due to charges: individual and configuration.
3. Capacitor as a storage device for charge and energy and its role in different circuits.
4. Ohm's law and Kirchhoff's laws for current distribution in a dc electric circuit.
5. Biot-Savart's law and calculation of magnetic field due to different current configuration.
6. Electromagnetic induction and calculation of induced emf.
7. To understand mathematical structure dealing with above-mentioned points.

Electrostatics

Problems

1. Three point charges q, 2q and 8q are placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of the system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges? [JEE 1987]

- This problem involves calculation of electric potential energy and electric field due to simple distribution of point charges.

  Tasks involved in this problem are:

  (a) To find number of possible ways in which three charges can be arranged along a straight line.

  (b) To calculate potential energy for each distribution, minimise potential and see which configuration gives minimum value of potential energy.

  (c) To calculate electric field at the site of 'q' for minimum configuration.

2. The distance between two positive charges q and 4q is '*l*'. How should a third charge Q be arranged for it to be in equilibrium? Under what conditions will the equilibrium of the charge Q be stable or unstable?

   - This problem involves balance (equality) of the forces due to two charges separated by a distance on the third charge.

     Tasks involved in this problem are:

   (a) To equate Coulomb's force due to two charges on the third charge and estimate the distance at which that happens.

   (b) To see that there are two solutions to a quadratic equation which in this case is not so obvious.

   (c) To understand that the solution for a point in between correspond to stable and outside correspond to unstable equilibrium.

3. Two fixed charges –2Q and Q are located at the points with co-ordinates (-3a, 0) and (+3a, 0) respectively in the xy-plane. (a) Show that all the points in the xy-plane where the electric potential due to the two charges is zero lies on a circle. Find its radius and the location of its centre. (b) Give the expression for potential V(x) at a general point on the x-axis and sketch the function V(x) on the whole x-axis. (c) If a particle of charge +q starts from rest at the centre of the circle, show by a short qualitative argument that the particle eventually crosses the circle. Find its speed when it does so. [JEE 1991]

   - This problem involves calculation of potential due to two charges in a plane. Finding locus of all the points at which potential is zero. Sketching the potential as a function of x. Seeing what happens to a charge at the centre of the circle.

     Tasks involved in this problem are:

   (a) To calculate potential as a function of (x,y) in a plane due to two charges.

   (b) To find locus of zero potential points.

   (c) To plot potential for points on x-axis.

   (d) To find out whether charge +q would cross the circle.

4. Two isolated metallic solid spheres of radii R and 2R are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin conducting wire. Find the new charge density on the bigger sphere. [JEE 1996]

   - This problem involves redistribution of charge till potential on the two surfaces become equal, and finding new distribution.

     Tasks involved in this problem are:

   (a) To find total charge and potential on each sphere.

(b) To decide criteria for distribution of charges when two spheres are connected by a conductor.

(c) To find new charge distribution.

5. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius r is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given charge Q, brought in contact with S_2, and removed. S_1 is recharged such that the charge on it is again Q; and it is again brought into contact with S_2 and removed. This procedure is repeated 'n' times. (a) Find the electrostatic energy of S_2 after n such contacts with S_1. (b) What is the limiting value of this energy as $n \rightarrow \infty$? [JEE 1998]

   - This problem involves generalisation of the process to large n value of what was done in problem 4. above.

   Tasks involved in this problem are:

   (a) To follow the procedure in problem 4. above repeatedly and see how it can be generalised for some 'n' trials.

   (b) To find what would happen after large number of steps.

6. Three concentric spherical metallic shells A, B and C of radius a, b and c (a < b < c) have surface charge densities σ, $-\sigma$ and σ, respectively. (i) Find the potential of the three shells A, B and C. (ii) If the shells A and C are at the same potential, obtain the relation between a, b and c. [JEE 1990]

   - This requires students to know how to calculate the potential at a point inside a sphere, outside and on the sphere due to surface charge.

   Tasks involved in this problem are:

   (a) To know that electric field inside a surface spherical charge distribution is zero and hence potential should be constant.

   (b) To add potential at each shell due to the each of the three shells.

   (c) To use condition given in part (ii) to get relation between (a), (b) and (c).

7. A 20 pF parallel plate capacitor with air as medium is charged to 200 V and then disconnected from the battery. What is the energy U_i of the capacitor? The plates are then slowly pulled apart (in a direction normal to the plate area) so that the plate separation is doubled. What is the mechanical work done in the process? What is the new energy U_f of the capacitor?

8. A 3μF parallel plate capacitor is connected to a battery of 400 V. The plates are then pulled apart as in P (7) above, so that the capacitance value becomes 1μF. This operation is carried out while the capacitor is still connected to the battery of 400 V. Calculate the mechanical work done. Account for the loss of energy of the capacitor.

9. In the problems P (7) and P (8) above what happens if dielectric slab or a metallic block is introduced instead of moving the plates.

   - In problem 7, charge is conserved and work has to be done to move capacitor plates apart against electrostatic attraction, which increases energy stored in the capacitor.
   - In problem 8, voltage remains constant as battery remains connected but capacitance and hence charge on the capacitor decreases. Reverse current flows and battery gets charged.

- Dielectric and metallic block both would be pulled in due to surface induced charges. In case of dielectric energy would increase due to increase in capacitance whereas in case of metallic plate if thickness were less than capacitor plate spacing would reduce effective distance between two plates and hence energy stored would increase.

Tasks involved in these problems are:

(a) To know how energy of a capacitor depends on C, Q and V.

(b) To know when to use charge and energy conservation.

(c) How does dielectric and conductor slab affect the geometry and charge or energy stored in the capacitor?

Electric Current

10. In the circuit shown in Fig. 1 the voltage measured across 2 K resistor was found to be 6 V, find it across 3 K. Find the resistance of the voltmeter. What would be the voltages measured if voltmeter was ideal?

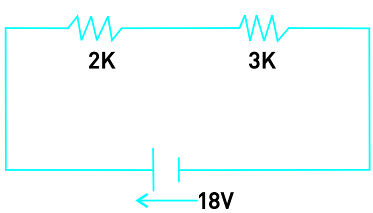


Fig. 1

- This problem makes students familiar with use of Ohm's law
- It conveys limitation of measuring device; in this case it is voltmeter and how its resistance affects measurement.

Tasks involved in this problem are:

a. To apply Ohm's law and Kirchhoff's law to the circuit.

b. To recognise contribution of voltmeter in the circuit due to its finite resistance.

11. In the circuit shown in Fig. 2 below, find current through each of the resistors. [Theraja]

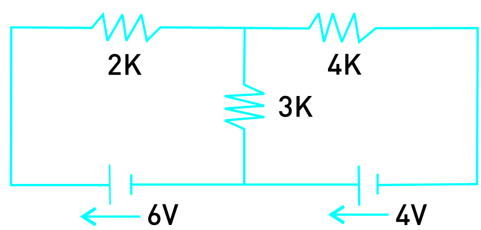


Fig. 2

- This problem involves application of Kirchhoff's laws for loop and junction to find current through each branch in the circuit.

Tasks involved in this problem are:

(a) To apply Kirchhoff's laws for loop (voltage) and junction (current).

(b) To solve the equations thus obtained to get current through each resistance.

- This problem is touchstone in the same sense as an inclined plane problem. It familiarises students with application of Kirchhoff's laws for loop and junction.

12. Twelve resistors each having resistance of value R are connected in the configuration of a skeleton cube. Referring to Fig. 3, find the effective resistance offered between points (i) A and F (ii) A and G (iii) A and B

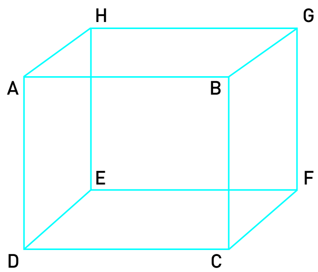


Fig. 3

- This problem shows the effectiveness of a network, familiarizes students with application of Kirchhoff's law and teaches how potential difference is path independent and how to exploit symmetry of the situation.

  Tasks involved in this problem are:

(a) To use symmetry to see how current gets distributed through different elements.

(b) To apply Kirchhoff's law (as an alternate method) to distribute current.

(c) To use the fact that potential difference between two points in a circuit is independent of the path chosen.

(d) To equate it to potential difference across effective resistance and hence evaluate the effective resistance.

Magnetism

13. A square loop of wire of edge 'a' carries a current *i*. Show that the value of B at the centre is given by, $B = 2\sqrt{2}\mu_o i/\pi a$. Also find magnetic induction at any point on the axis.

14. A wire in the form of a regular polygon of n sides is just enclosed by a circle of radius 'a'. If the current in this wire is '*i*', show that the magnetic induction at the center of the circle is given by $B = (\mu_o ni/2\pi a) \times \tan(\pi/n)$. Show that as $n \rightarrow \infty$ this result approaches that of a circular loop.

- Problems 13 and 14 involve application of formula arrived at for magnetic field due to a current-carrying wire of finite length.
- Problem 14 involves generalisation to n-sided polygon and checking if the result matches with circle if 'n' is large.

  Tasks involved in this problem are:

(a) To find angles subtended by straight conductors of finite length at the point (centre of a regular polygon).

(b) To calculate magnetic field due to one such side and hence 'n' sides.

(c) To let 'n' be very large and see if result reduces to that of a circle.

15. Find magnetic field at any point on the axis of a circular current carrying loop.

- This problem involves calculating magnetic field on the axis of a circular loop using Biot-Savart's law.

  Tasks involved in this problem are:

(a) To apply Bio-Savart's law to a current - carrying loop.

(b) To work out direction of field due to diametrically opposite elements.

(c) To find out which component contributes and which one gets neutralised.

(d) To integrate to final value of B-field.

16. Current flows around the cubical wire frame in the figure given. What is the direction and magnitude of the magnetic field at the centre

of the cube? [Hint: you may find it useful to employ superposition principle.][InPhO 2004]

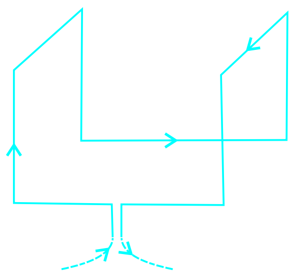


Fig. 4: Skeleton Wire

- This problem is an extension of problem 13 but in 3-dimension but can be easily solved if one focuses on symmetry consideration.

  Tasks involved in this problem are:

(a) To recognise what happens if missing wires were there.

(b) To recognise that missing wires put back effectively adds nothing to the problem but facilitates viewing as combination of squares.

(c) To find magnetic field of a square loop at a point on its axis and superpose all such contributions vectorally.

Electromagnetic Induction

17. A metal rod OA of mass m and length r is rotating with a constant angular speed w in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform and constant magnetic induction **B** is applied perpendicular and into the plane of rotation as shown in Fig. 5. An inductor L and an external resistance R are connected through a switch S between the point O and a point C on the ring to form an electric circuit. Neglect the resistance of the ring and the rod. Initially the switch is open.

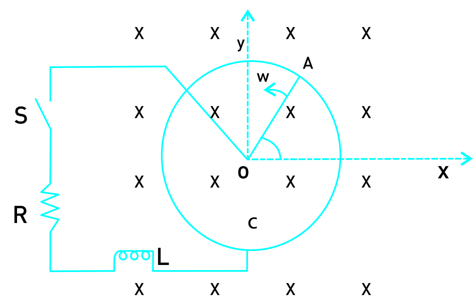


Fig. 5

(a) What is the induced emf across the terminals of the switch?

(b) The switch S is closed at time t = 0.

  (i) Obtain an expression for the current as a function of time.

  (ii) In the steady state, obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at t = 0. [JEE 1995]

- This problem requires students to use Faraday's laws and Lenz's law to find induced emf and induced current in the rod.
- The problem also involves working of an LR circuit and effect of gravity on the rotating rod.

  Tasks involved in this problem are:

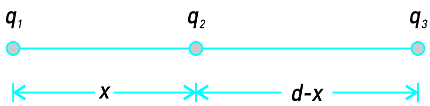
(a) To calculate induced emf using Faraday's laws and Lenz's Law.

(b) To find current knowing the fact that given circuit is an LR circuit.

c. To incorporate the fact that rod is in vertical plane and hence is under influence of gravity and calculate the torque needed for constant angular speed.

Solutions

1. Electrostatic Field and Potential



$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{x} + \frac{q_2q_3}{d} + \frac{q_3q_1}{(d-x)}\right)$$

For potential to be minimum :

$$\frac{\partial V}{\partial x} = 0 \Rightarrow x = \frac{d}{1 \pm \sqrt{q_1/q_3}}$$

: negative solution ruled out (as it means $x > d$)

q_1	q_2	q_3	x	$V/(1/4\pi\varepsilon_0)$
q	$2q$	$8q$	6.65	$7.998q^2$
$2q$	q	$8q$	3.0	$3.778q^2$
q	$8q$	$2q$	5.27	$6.029q^2$

2nd arrangement indicates the minimum configuration.

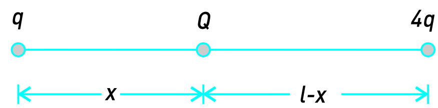
Alternatively

Since potential energy depends on $q_1 q_2$, large charges should be kept apart. This also gives 2nd arrangement as mentioned. Substituting all the values: $d = 9$ and $q_1 = 2q$ and $q_3 = 8q \Rightarrow x = 3$ cm. Electric field in this situation at q due to the other two will be:

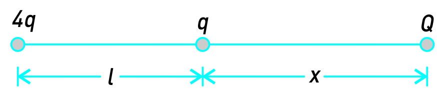
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2q}{x^2}\hat{i} - \frac{8q}{(d-x)^2}\hat{i}\right) = 0$$

2. Static Equillibrium

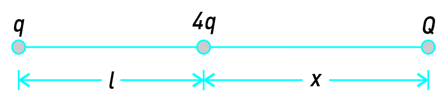
Arrangement 1



Arrangement 2



Arrangement 3



Naturally it is only 1st arrangement that can be equilibrium as 2nd and 3rd arrangement means same direction of force due to each charge.

In 1st arrangement : $\frac{qQ}{x^2} = \frac{4qQ}{(l-x)^2}$ equating the two forces.

$\therefore (l-x)^2 = 4x^2 \Rightarrow l - x = + 2x$

$\therefore x = l/3$ or $x = -l$ the second solution is ruled out any way.

Thus at $x = l/3$ charge will be stable (along the line of charges) irrespective of sign of Q.

3. Electric Potential

(a) $$V(x,y) = \frac{1}{4\pi\varepsilon_0}\left(\frac{-2Q}{\left[(x+3a)^2+y^2\right]^{1/2}} + \frac{Q}{\left[(x-3a)^2+y^2\right]^{1/2}}\right)$$

$V(x, y) = 0 \Rightarrow 4[(x-3a)^2+y^2] = [(x+3a)^2+y^2]$

Which gives :

$x^2 + y^2 - 10ax + 9a^2 = 0$: Circle with centre : $(5a, 0)$ and radius $r = 4a$

(b) On x – axis : y = 0

$$\therefore V(x) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{-2}{|x+3a|} + \frac{1}{|x-3a|}\right]$$

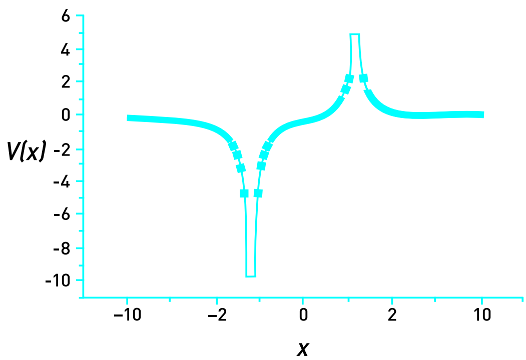


Fig. 6

(c) For q at $(5a,0)$: force

$$F = \frac{qQ}{4\pi\varepsilon_0}\left\{\frac{1}{(2a)^2} - \frac{1}{(8a)^2}\right\} = \frac{7}{32}\frac{qQ}{4\pi\varepsilon_0 a^2}$$

in positive x-direction.

K.E. at $(x = 9a)$ + P.E. at $(x = 9a)$ = P.E. at $(x = 5a)$

$$\Rightarrow v = \left[\frac{qQ}{4\pi\varepsilon_0(2ma)}\right]^{1/2}$$

4. Electrostatics and Surface Charge Distribution

$Q_1 = 4\pi R^2\sigma$ and $Q_2 = 4\pi(2R)^2\sigma$

$V_1 = Q_1/4\pi\varepsilon_0 R = \sigma R/\varepsilon_0$ and similarly $V_2 = \sigma(2R)/\varepsilon_0$

When the two spheres are connected, charge transfer takes place till both the potentials become equal such that total charge is conserved.

$Q_1 + Q_2 = Q_1' + Q_2'$ with σ_1 and σ_2, respectively, such that $V_1' = V_2'$.

This gives $\sigma_1 + 4\sigma_2 = 5\sigma \Rightarrow \sigma_1 = 5/3\sigma$ and $\sigma_2 = 5/6\sigma$

5. Electrostatics and Surface Charge Distribution

Step I: $Q = q + q_1$ (here q is the charge on S_1 and q_1 is charge on S_2 after first contact)

and $q/r = q_1/R \Rightarrow q_1 = Q\left(\frac{R}{R+r}\right)$

Step II: $Q + q_1 = q' + q_2$ (here q' is the charge on S_1 and q_2 is charge on S_2 after second contact)

and $\frac{(Q+q_1) - q_2}{r} = \frac{q_2}{R}$

$$\Rightarrow q_2 = (Q+q_1)\left(\frac{R}{R+r}\right) = Q\left\{\left(\frac{R}{R+r}\right) + \left(\frac{R}{R+r}\right)^2\right\}$$

repeating the procedure gives the nth step:

$$q_n = Q\left\{\left(\frac{R}{R+r}\right) + \left(\frac{R}{R+r}\right)^2 + \ldots + \left(\frac{R}{R+r}\right)^n\right\}$$

$$\text{(a) } U_n \frac{1}{2}\frac{q_n^2}{4\pi\varepsilon_0 R} = \frac{Q^2}{8\pi\varepsilon_0 R}\left\{x\left(\frac{1-x^n}{1-x}\right)\right\}^2$$

where $x = \left(\frac{R}{R+r}\right)$

(b) as $n \to \infty$ $x^n \to 0$

$$\therefore U_{n\to\infty} = \frac{Q^2}{8\pi\varepsilon_0 R}\left(\frac{x}{1-x}\right)^2 = \frac{Q^2}{8\pi\varepsilon_0}\frac{R}{r^2}$$

6. Electrostatics

(i) Potential at A due to B and C :

$$\frac{1}{4\pi\varepsilon_0}\left[\frac{-\sigma.4\pi b^2}{b}+\frac{\sigma.4\pi c^2}{c}\right]$$

due to A itself : $\frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma.4\pi a^2}{a}\right]$

$$\therefore V_A=\frac{\sigma}{\varepsilon_0}(a-b+c)$$

Similarly : $V_B=\frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{b}-b+c\right)$ and

$$V_C=\frac{\sigma}{\varepsilon_0}\left(\frac{a^2}{c}-\frac{b}{c}+c\right)$$

(ii) $V_A = V_C \Rightarrow a - b + c = (a^2 - b^2)/c + c$

$\therefore c = a + b$

7. Electrostatics: Capacitance

$U_i = ½ CV^2 = 4\times10^{-7}$ J

Since plates are disconnected from the batteries, Q remains constant and hence $U = (½)(Q^2/C)$ where $C = \varepsilon_0 A/d$ and since d is doubled, C gets halved and $U_f = 2U_i$

The additional energy comes from the work done in moving plates apart. Thus work done $W = U_i = 4\times10^{-7}$ J and $U_f = 8\times10^{-7}$ J

8. Electrostatics: Capacitance

As p.d. across capacitor plates remain constant, mechanical work done is zero.

$\Delta U = \Delta(½ CV^2) = ½ (\Delta C) V^2 = -0.16$ J

The loss of energy indicate that energy returned to the battery.

9. Electrostatics: Capacitance

In Fig. 7 if dielectric or metallic block is introduced, they effectively increase capacitance. However, charge on the plate remains same. Due to induced charges on the surface of block, it will be pulled inside. Capacitor will do some work in pulling. This would reduce energy stored in the capacitor.

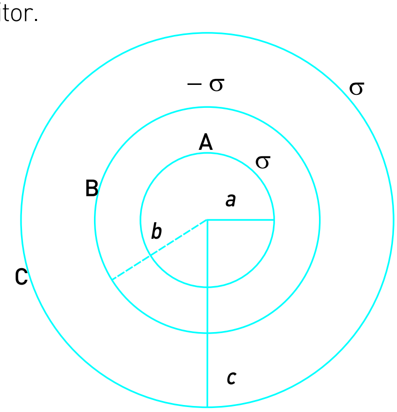


Fig. 7

In Fig. 8 too if dielectric or metallic block is introduced, they effectively increase capacitance. However, this time voltage difference across the plates remains same. Capacitor will still do some work in pulling, but more charge would flow to the plates from the battery which also provides the additional energy that is (i) stored in the capacitor and (ii) used for doing work.

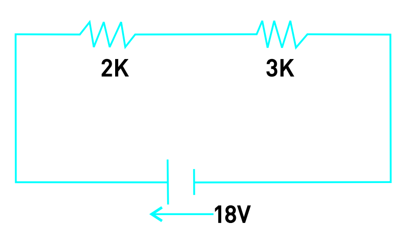


Fig. 8

10. Electric Current: Ohm's Law

Let resistance of voltmeter be R. $2R/(2 + R)$ and $3K$ divides voltage in the ratio $1 : 2$

$\therefore 2R/(2 + R) = 3/2 \Rightarrow R = 6\ K$.

Thus across $3\ K$, effective resistance will be $2\ K$. Thus voltage measured will be $9\ V$.

An ideal voltmeter would measure these voltages to be $7.2\ V$ and $11.8\ V$.

11. Electric Current: Kirchhoff's Laws

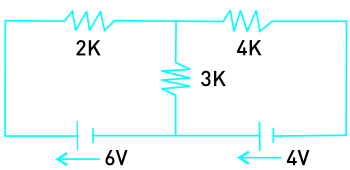


Fig. 9

Let $E_1 = 6\ V$ and $E_2 = 4\ V$

Let current through $2\ K$, $3\ K$ and $4\ K$ be i_2, i_3 and i_4, respectively.

According to Kirchhoff's law for junction (current) : $i_2 = i_3 + i_4$

According to Kirchhoff's law for loop (voltage) : $E_1 = 2i_2 + 3i_3$ and $E_2 = -3i_3 + 4i_4$

Solving which we get : $i_2 = 27/13\ mA$, $i_3 = 8/13\ mA$ and $i_4 = 19/13\ mA$

12. Effective Resistance

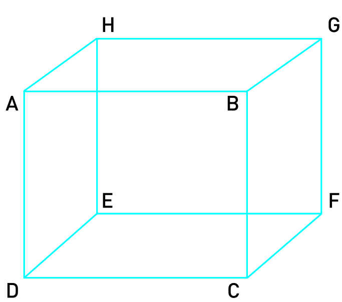


Fig. 10

This problem involves dividing current i at entry point using symmetry and combining at exit point. Voltage drop between the two points to be calculated along any chosen path and to be equated to ix. Where x is the effective resistance to be calculated.

Ans: (i) 5R/6 (ii) 3R/4 and (iii) 7R/12

13. Magnetism

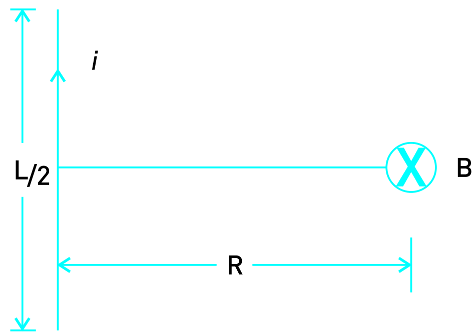


Fig. 11

The magnitude of the magnetic field at a distance R from a conductor of length L carrying current i

is given by $B = \frac{\mu_0 i}{4\pi R} \frac{L}{\sqrt{R^2 + L^2/4}}$

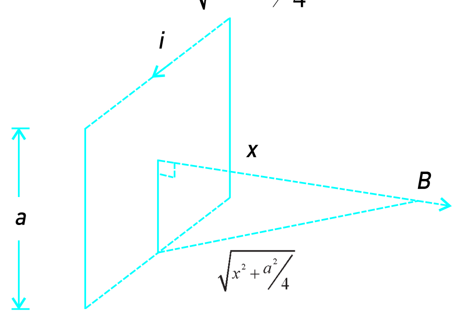


Fig. 12

Refering to the Fig. 12, magnetic field at a point on the axis at a distance x from the center of the square loop of size *a* is given by

$$B = \frac{\mu_0 i a}{\pi\sqrt{x^2 + a^2/4}\sqrt{x^2 + a^2/2}}$$

Thus at the centre of the square loop

$$B = \frac{2\sqrt{2}\mu_0 i}{\pi a}$$

14. Magnetism

For a polygon of *n* sides inside a circle of radius *a* : in the formula in above problem *L/2*→ *a sin (π/n)* and *R*→ *a cos (π/n)* which gives $B = \frac{n\mu_0 i}{2\pi a}\tan\frac{\pi}{n}$

Thus as $n \to \infty$: $B = \frac{\mu_0 i}{2\pi a}$ same as result known for circular loop.

15. Magnetic Field at any Point on the Axis of a Circular Current-Carrying Loop

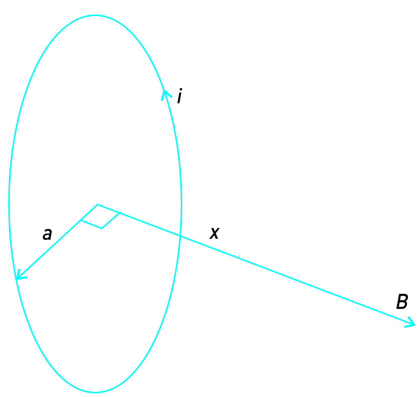


Fig. 13

Using Biot-Savart's law it can be shown that component perpendicular to axis cancels that due to diametrically opposite element and parallel component adds up. Thus resultant field works out to be $B = \frac{\mu_0 i a}{2(a^2 + x^2)^{3/2}}$

16. Magnetic Field at Centre of the Skeleton Cube

The straight forward application of Biot-Savart's law for six straight conductors of finite length (taking care of directions gives : $B = \frac{2\mu_0 i}{\sqrt{3}\pi a}$

The problem can also be viewed as entire cube : the missing sides added would contribute zero current any way.

17. Electromagnetic Induction

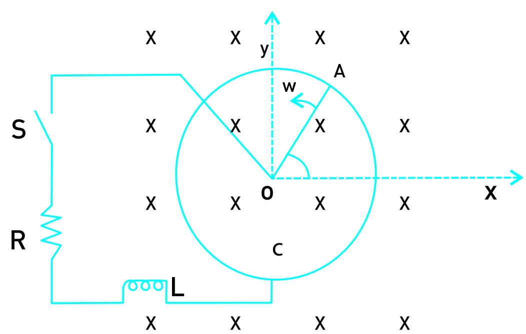


Fig. 14

(a) $E_{induced} = -\frac{d\phi}{dt} = -\frac{d}{dt}(BA) = -B\frac{d}{dt}(A) = -B\frac{d}{dt}(\frac{1}{2}r^2\theta) = -\frac{1}{2}Br^2\omega$

(b) (i) $I = \frac{E}{R}(1 - e^{-Rt/L})$

(ii) $I_{steady} = E/R$ as $t \to \infty$ ∴ $I_{steady} = \frac{Br^2\omega}{2R}$

$\tau_1\omega = P = I_s^2 R = \frac{B^2 r^4 \omega^2}{4R}$: dissipating across *R*

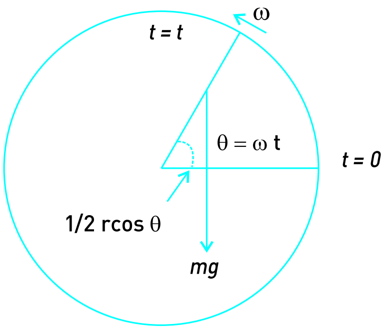


Fig. 15

$$\tau_2 = \frac{mgr}{2}\cos\theta = \frac{1}{2} mgr\cos\omega t$$: against gravity

$$\therefore \tau = \tau_1 + \tau_2 = \frac{B^2 r^4 \omega^2}{4R} + \frac{1}{2} mgr\cos\omega t$$

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