HOW BIG IS THE MOON AND HOW FAR IS THE SKY[1]?

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It is a matter of common observation that the Sun, the Moon and constellations appear much bigger when near the horizon than when they are high up in the sky[1]. The Sun and the Moon when high up appear (to most people) a little less than a foot in diameter, and near the horizon they look from 2 to 3 times bigger – the effect is greater in twilight and when the sky is clouded. It is needless to add that the entire phenomenon is psychological for there is no physical reason why we should associate a linear size of one foot with an angle of about half a degree – the angle subtended by the Sun's diameter is 31' 59" and the mean angle[2] for the Moon's diameter is 31' 5', and why the size should appear to vary with altitude though the angle subtended at the eye and hence the size of the retinal image remains constant.

The apparent variation of size with altitude exists also in the after-image of the Sun (and also the Moon) which is obtained by viewing the Sun for an instant and then blanking. The after-image of the Sun at the horizon as background appears to be of the same size as the Sun, but is reduced to about half its size when projected on the sky near the zenith. If instead of projecting on the sky the after-image of the Sun when at horizon, we project it on a wall, then it appears smaller than the Sun if the distance of the wall is less than about 200 feet, but on a wall at about 200 feet or beyond the size appears to be the same as that of the Sun. This shows that the distance of the horizon-sky appears to be about 200 feet, and of the sky at zenith about half of this[3].

There seems to be a possible connection between the apparent variation of the size of heavenly bodies with the altitude and the apparent flattening of the vault of heaven.

A very interesting article by Professor H.N. Russell had appeared (Scientific American, Oct., 1940) on the subject of apparent variation in the size of the Moon. Also see Hargreaves, observatory, June, 1940.

[1]The apparent variation of size persists even when the bodies are seen through a telescope.

[2]When the Moon is at the horizon its distance from the observer is greater by the earth's radius than when it is at the observer's zenith, and therefore the angular diameter, at the horizon compared to that the at the zenith is actually smaller by 0.5′. The Variation in the distance of the Moon from the earth due to the eccentricity of its orbit introduces in the angular diameter a variation of over 10 per cent.

[3]200 feet is a little less than one-third the radius of stereoscopic vision calculated on the basis of one minute as the resolving power of the eye.

When we look at the sky, the impression that we get is not that of an inverted hemisphere with ourselves at its centre, but it appears like a flattened dome whose distance from eye to zenith is smaller than the distance from eye to horizon, the ratio being from 2 to 4 depending on the observer and the circumstances attending the observation. The apparent flattening of the sky is felt vividly when try to locate the mid-point of the arc joining the zenith and the horizon.

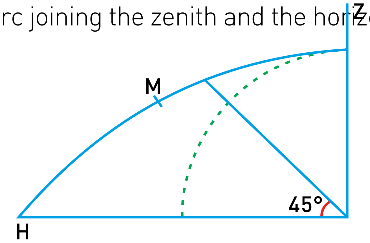

Fig.1. H is the horizon and Z is the zenith. M is the mid-point of the arc of the sky HZ.

If we point our hand or a stick in the direction of the mid-point, and if then the angle be measured, it is found that the altitude of the estimated mid-point M is much lower that 45°. It generally lies between 20 ° and 30 ° (Fig.1). In these and other psychological observations it is necessary that the observer should allow himself to get the impression as he sees or feels it, and not modify it by making a conscious effort in trying to see what he (according to his preconceived ideas or theoretical knowledge) ought to have seen. We are observing an illusion and the observation is vitiated to the extent that we make any conscious effort to overcome it. Scientific observation and study of illusion require psychological training.

Robert Smith[4] (Optics, 1738) suggested more than two centuries ago that we imagine the Moon, the Sun and the stars to be at the same distance as the sky, and therefore they appear to be several times more distant when at the horizon than at zenith: and as the angle subtended by than at the observer's eye remains the same, greater distance is associated with a proportionate increased (linear) size. In support of this view it may be noted that in twilight or when it cloudy the sky looks more flattened and therefore at the horizon farther away than ordinarily, and the Sun or the Moon at the horizon also appear larger. But why does the sky appear flattened? Let us first take up an interesting explanation due to Sterneck[5], which, however, as will appear later has to be discarded.

Sterneck gave an empirical relation between the true distance and the apparent distance of an object, and he was able to connect in this way a large number of phenomena, e.g., street-lamps father away than about 150 yards seem at night to be all at the same distance; rectangular fields seen from a train appear trapezia; the steepness of a mountain-slope is over-estimated when seen from the bottom of the mountain and under-estimated when we stand at the top; and the flattening of the celestial vault. Van Sternieck's formula is

$$x' = \frac{cx}{c+x} \qquad \text{..........} (1)$$

where x is the true distance, x ' the apparent distance and c is a constant. The apparent distance is always smaller than the true

[4]M. Luckiesh, Visual Illusions, D. Van Nostrand Co., New York, (1922), Chapter XI.

[5]M. Minnaert, Light and Colour in the Open Air, Bell and Sons, London, (1940). Chapter IX. This is one of the best books on "everyday physics" that the writer has come across.

distance, and c is the limit which it approaches for increasing true distance. The value of c ranges from about 100 yards to 10 miles depending upon the nature of the object whose distance is estimated and on the circumstances under which it is observed.

When the sky is clouded, the clouds, being at an extremely small height compared to the earth's radius, form a practically flat ceiling above us[6]. The distance r between the observer and the cloud in the direction θ from the vertical is p sec θ, p being the vertical height of the cloud, and therefore if δ denotes the ratio of the apparent distance in the direction θ to the apparent vertical height of the cloud, then from equation (I), δ will be given by

$$d = \frac{1+\frac{c}{p}}{1+\frac{c}{p}\cos q} = \frac{d_0}{1+(d_0-1)\cos q} \qquad \text{......... 2}$$

where δ_0 is the ratio of the apparent horizon-distance to zenith-distance. The cloudy sky should therefore appear like a hyperboloid of revolution (with the observer at its focus), which does agree with our general impression of it.

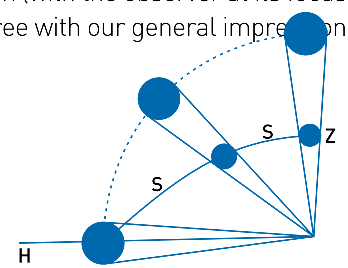

Fig. 2. H is the horizon, Z is the zenith, S.S. represents the sky. The figure illustrates Robert Smith's explanation of the apparent variation in the size of the Sun (or the Moon) due to the apparent flattening of the sky.

However, not only a cloudy sky, but a blue and a starry sky also give the same impression of being flattened – only the flattening is less, and it is difficult to see how Van Sterneck's explanation could be applied to a featureless blue sky. But a serious objection to this explanation is the fact that the apparent shape of the sky is dependent on the way the observer holds his body during the observation. If instead of standing, the observer lies flat on is back on the ground, the appearance of the sky is completely altered –.it is spherical towards his feet but compressed towards his head. The flattening of the sky is relative to the observer's "personal horizon" which is a great circle perpendicular to his backbone. When the head is held in its normal position relative to the body, the observer's gaze is towards his personal horizon. The head has to be thrown backwards to see the sky above the personal horizon, and bent forward to see below it. The sky below the personal horizon appears spherical and flattended above it. In fact, if an observer supports himself from a horizontal bar with the body vertical and head downwards, the whole sky is below his personal horizon an appears to him spherical[7].

[6] If the cloud be at vertical height of one mile, then, even for an altitude of 100, the distance of the cloud, assuming the cloud-bank to be a flat ceiling, will exceed the distance calculated on the assumption that the cloud-bank is a concentric sphere round the earth by less than 0.5 per cent.

[7] M. Minnaert, loc, cit., p.163 Fig 3 is also taken from this book.

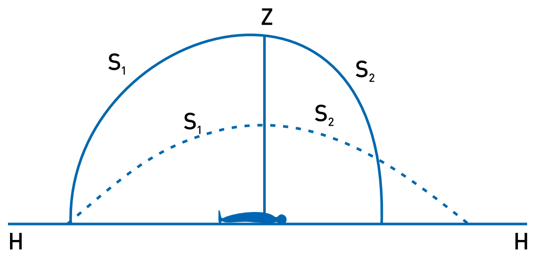

Fig. 3: H is the horizon and Z is the zenith. S_1 S_1 is the apparent shape of the sky when the observer is standing. S_2 S_2 is the apparent shape of the sky when the observer is lying on his back.

It is not only the flattening of the sky which is related to the personal horizon, but the apparent variation in the size of heavenly bodies is also dependent on it. This has been conclusively established by Professor Boring[8] (and his colleagues) at Harvard, who recently reported his results to the United States National Academy of Sciences. The Moon looks big when it is near the observer's personal horizon. It appears smaller when it is away form the personal horizon, it being immaterial whether it is above or below it – the phenomenon is symmetrical with respect to the personal horizon. Further, the Harvard psychologists find that the Moon, even when high in the sky, reduces the impression of looking (to an observer standing on the ground) as big as a disk about 5 inches in diameter placed 10 or 12 feet away – the apparent angular diameter is about four times its real value. At the horizon the Moon appears twice a big. The apparent magnification of the angular diameter is significant: our estimates of the size and distance are not conditioned by the actual visual angle.

The question remains: why the Moon looks largest when it is at the personal horizon? Why there is an association between the apparent size of the Moon and the bending of the head (backwards of forwards depending on whether the Moon is above or below the personal horizon) necessary to look at it? A possible explanation[9] is to be sought in our everyday experience. When an object approaches us we have in most circumstances to bend our head to see it, – forwards if the object is on the ground, and backwards if it is a flying bird or a cloud. The angle subtended by the object increases with increasing bending of the head, but, provided the object was approaching us from not too large a distance, our training in interpreting visual perceptions has been such that the impression of its size remains almost the same: it is nearly its true size. It seems that we assign unvarying size not by making allowance for the varying distance, but on the contrary we carry as it were the 'size' in us (' we are geometricians by nature') and judge of distance from the visual angle through its (size) help. We get so much accustomed from common experience to a large visual angle when the object is seen with the head in its normal position and to a small visual angle when the same object is seen with a bent head – the impression of size being the same in the two cases –that for the Moon, as the angle

[8] H.N. Russell, loc. Cit.

[9] This has been suggested by Professor Ruessell; loc.cit. See also Minnaert, p. 162.

[10] It seems that a forward bending of the head has no effect on our estimate of distance, but bending the head backwards produces a bias in favour of underestimating the distance. The value of c in Sterneck's formula is unaffected in the former case, but reduced in the latter case.

remains the same, we get an impression of a smaller size when we have to bend our head in order to see it.

The apparent shape of the sky can also be explained on similar lines by assuming that our daily experience accustoms us to a relation between distance of objects and the bending of the head necessary to see them, but it must be admitted that these are only plausible suggestions and at present no explanation of the apparent variation in the size of heavenly bodies or the shape of the sky can be regarded as reasonably satisfactorily established[10].

We have mentioned that if the distance of an object is not too large, the impression of its size is independent of the distance. When the distance is large, the apparent size decreases, and it appears very likely that the relation between apparent size and real size is of the same form as Sterneck's formula for apparent dis

$$y' = \frac{yc'}{c'+x'}$$

or

$$y = y' + \frac{y'}{c'}x \qquad (3)$$

where y' is the apparent size of an objet of true size y and at a distance x,c' is a constant, its particular value depending on the circumstances under which the object is observed and is probably different from value in the distance-formula (1). The apparent size is half the true size for x = c' . So long as x is small compared to c, the apparent size does not vary appreciably, it is almost the same as the true size. For x large compared to c', the apparent size is inversely proportional to x. Two straight lines (telegraph wires, long stretched strings, straight edges of a foot-path etc.) will appear when they are not actually parallel, but the distance between them increases linearly with x so as to satisfy (3). Relation (3) should hold fairly accurately for terrestrial objects.

When we look at an object through a telescope or binoculars, the visual angle subtended at the eye is increased by a factor which is the magnifying power (m) of the instrument. The effect is the same as if the true distance of the object had been reduced m times, and the apparent size of the object as seen through the telescope will be

$$y' = \frac{yc'}{c'+x/m} = \frac{myc'}{mc'+x'} \qquad ....(4)$$

For x small compared to c' m, the apparent size will be almost the same as the true size. This seems to agree with our (qualitative) experience and a detailed investigation will be interesting.

It may be remarked that our judgement of speed say, when we are sitting in a car, is also modified because of the under-estimation of distance. If we judge the speed by looking at an object at a distance x from us, then the true speed $\left(\frac{dx}{dt}\right)$ and the apparent speed $\left(\frac{dx'}{dt}\right)$ are connected by

The article written by late Prof. D. S. Kothari was published in Science and Culture in 1941. Professor Kothari was teaching Physics in the University of Delhi at that time. Prof.Kothari played a significant role in the development of science and technology in post-Independence era. He acted as Scientific Advisor to the Government of India, Vice Chairman, UGC and as Chancellor of the Jawaharlal University.

the relation $\left(\frac{dx'}{dt}\right)=\left(\frac{c}{c+x}\right)^2\frac{dx}{dt}$, and if we are looking through binoculars, it becomes $\frac{dx'}{dt}=\frac{1}{m}\left(\frac{mc}{mc+x}\right)^2\frac{dx}{dt}$ or for x small compared to mc, the apparent speed is l/mth of the true speed.