THE PRINCIPLE AND THE METHOD OF FINDING OUT THE CUBE ROOT OF ANY NUMBER

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The two methods usually employed to find the cube root of a number are, (i) the method of factorisation, and (ii) the division method. However, it is the method of factorisation that generally finds a place in the school mathematics to obtain the cube root of a number. It may be pointed that this method is convenient to find cube root of the numbers that are a perfect cubes. Here an attempt has been made to explain the method to find the cube root of any number by the method of division. The advantage of this method over the method of factorisation is that it is possible to find the cube root of any number up to the desired decimal places.

Section (A)

The principle of finding out cube root of a number is based on the following known identities and the pattern:

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$= a^3 + (3a^2 + 3ab + b^2)b$ (i)

and $(a + b + c)^3$
$= (a + b)^3 + 3(a + b)^2c + 3(a + b)c^2 + c^3$

$= a^3 + (3a^2 + 3ab + b^2)b + [3(a + b)^2 + 3(a + b)c + c^2]c$ (ii)

also $(a + b + c + d)^3$
$= (a + b + c)^3 + 3(a + b + c)^2d + 3(a + b + c)d^2 + d^3$

$= a^3 + (3a^2 + 3ab + b^2)b + [3(a + b)^2 + 3(a + b)c + c^2]c + [3(a + b + c)^2 + 3(a + b + c)d + d^2]d$ (iii)

on the above pattern, we can write

$(a + b + c + d)^3$

$= a^3 + (3a^2 + 3ab + b^2)b + [3(a + b)^2 + 3(a + b)c + c^2]c + [3(a + b + c)^2 + 3(a + b + c)d + d^2]d + [3(a + b + c + d)^2 + 3(a + b + c + d)e + e^2]e$ (iv)

We see that the identities (i), (ii), (iii) and (iv) have a pattern. Therefore with the help of this pattern we can write cube of any expression.

Section (B)

We can suppose a number in the form of $(a + b)^3$ or $(a + b + c)^3$ or $(a + b + c + d)^3$ etc., and then we find out the value of (a + b), (a + b + c) or (a + b + c + d) as the case be. Thus, the cube root of that number is determined.

In (a + b), a is tens and b is ones.

or if a is ones, then b will be first digit after decimal point.

or if a is first digit after decimal point, then b will be second. In (a + b + c), a is hundreds, b is tens and c is ones.

or if a is tens, then b is ones and c is first digit after decimal point.

or if a is ones, then b and c are first and second digits after decimal points, etc.

Section (C)

We know that $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$ and $5^3 = 125$ .... and $10^3 = 1000$, $20^3 = 8000$, $30^3 = 27000$, $40^3 = 64000$ and $100^3 = 1000000$, $200^3 = 8000000$, $300^3 = 27000000$, $400^3 = 64000000$.

It shows that cube root of a number having 1, 2 or 3 digits will be of one digit.

Cube root of a number, having 4 to 6 digits, will be of two digits.

Cube root of a number, having 7 to 9 digits, will be of three digits.

Section (D)

If a number is multiplied by 1000, then its cube root will be increased by ten times.

If a number is multiplied by 1000000, then its cube root will be increased by hundred times.

If a number is multiplied by 10^9, then its cube root will be increased by thousand times. And so on.

With the help of this property, we can find out cube root of small numbers. This property may also be helpful in finding cube root upto desired number of decimal places.

Section (E)

As it has been already said, Section (B) above that a number can be written in the form of $(a + b)^3$, $(a + b + c)^3$ or $(a + b + c + d)^3$, etc. i.e., in the form of

$a^3 + (3a^2 + 3ab + b^2)b$ (I)

or

$a^3 + (3a^2 + 3ab + b^2)b + [3(a + b)^3 + 3(a + b)c + c^2]c$ (II)

or $a^3 + (3a^2 + 3ab + b^2)b + [3(a + b)^2 + 3(a + b)c + c^2]c + [3(a + b + c)^2 + 3(a + b + c)d + d^2]d$ (III)

Procedure of Finding Cube Root

(I) We can easily find the value of a as in Section (C). We subtract a^3 from expression (I) above.

The remainder is $(3a^2 + 3ab + b^2)b$

(II) Now putting the value of a in the expression $(3a^2 + 3ab + b^2)$, we try to find out the value of b by division method through approximation.

Now value of a and b are known.

(III) We put the values of a and b in expression $[3(a + b)^2 + 3(a + b)c + c^2]$ and we try to find out value of c by division method through approximation.

This process can be continued for finding values of d and e etc.

Solved Examples

Example 1

To find out the cube root of 3375.

Method

(i) We know $10^3 = 1000$, $1000 < 3375$ and $20^3 = 8000$, $8000 > 3375$

It shows that the cube root of 3375 lies between $3.10^2 + 3.10.5 + 5^2$

10 and 20, therefore $a = 10$ and $a^3 = 1000 = 475$

	10+5
10^2	3375
	1000
	2375
	2375
	0

(ii) Now subtracting 1000 ($10^2 \times 10$) from 3375 remainder is 2375 Cube root of

∴ 3375 = 10 + 5 = 15

(iii) To find the value of b we put a = 10 in expression $3a^2 + 3ab + b^2$. We get

$= 3.10^2 + 3.10$ app.

= 330 app.

(iv) If we divide 2375 by 330 we get quotient = 7 app. i.e. b = 7. Putting a = 10 and b = 7 in expression

$(3a^2 + 3ab + b^2)b$, we get $(3.10^2 + 3.10.7 + 7^2)7$

$= (300 + 210 + 49)7 = 459 \times 7$

= 3913

(v) We see that 3913 > 3375

So we put a = 10 and b = 5 in the expression
We get $(3.10^2 + 3.10.5 + 25)5$

$= (300 + 150 + 25)5 = 475 \times 5 = 2375$

Therefore b = 5 is the right value.

Cube root of 3375 = 15.

Example 2

Find cube root of 54321

(i) Making groups of three from RHS, we see that 54 is left whose cube root lies between 30 and 40.

So a = 30 and $a^3 = 27000$

Subtracting 27000 from 54321, remainder is 27321.

(ii) Putting a = 30 in exp. $3a^2 + 3ab + b^2$

Value of expression $= 3.30^2 + 3.30$

$= 2700 + 90$

= 2790 app.

Dividing 27321 by 2790,
we see that quotient may be 8 or 9
Putting a = 30 and b = 8 in the expression

$(3a^2 + 3ab + b^2)b$, we get

$(3.30^2 + 3.30.8 + 8^2)8$

$= (2700 + 720 + 64)8$

= 27872   27321

Therefore b = 7 is the correct value

Put b = 7 in the expression

We get $(3.30^2 + 3.30.7 + 7^2)7$

$= 3379 \times 7$

= 23653

Subtracting 23653 from 27321, remainder is 3668

$(3.30^2 + 3.30.7 + 7^2)$

=3379

$$\begin{bmatrix} 3(300+70)^2 \\ +3(300+70)(8)+(8^2) \end{bmatrix}$$

= 419644

	30+7+.8
30^2	54321
	27000
	27321
	23653
	3668000
	3357152
	310848

(iii) We increase three zeros in remainder 3668 to find out cube root in decimal. Here a, and b will increase 10 times.

i.e., a = 300 and b = 70

Put a = 300 and b = 70 in expression

$3(a + b)^2 + 3(a + b)c + c2$. We get $3(300 + 70)2 +$ $3(300 + 70)$

$= 3.(370)^2 + 3(370)$

$= 410700 + 1110$

= 411810

Dividing 3668000 by 411810.
Quotient is 8 or 9

Putting a = 300, b = 70 and c = 8 in the expression $[3(a + b)2 + 3(a + b)c + c^2]c$

We get $[3(300 + 70)^2 + 39300 + 70) (8) + (8^2)](8)$

$= (410700) + (8880 + 64) (8)$

$= 419644 \times 8$

$= 335715 < 3668000$

Therefore c = 8 is correct value.

Cube root of 54321 = 37.8

Note: If we want to find out cube root up to 2 decimal places, we will have to continue the process increasing three zeros in the remainder. Now a, b, c will increase 100 times i.e., a = 3000, b = 700 and c = 80.

Example 3

Find cube root of 15

	2.46
2^2	15
	8
	7000
	5824
	1176000
	1062936
	113064

(i) Cube root of 15 lies between 2 and 3 therefore a = 2 and $a^3 = 8$

ii) Subtracting 8 from 15 remainder is 7. We increase 3 zeroes to find first decimal value. Now a will become 10 times i.e.,

$[3.20^2 + 3.20.4 + 16]$

i.e., a = 20 $\qquad = 1456$

(iii) Putting a=20 in the exp. $3a^2+3ab+b^2$, we get $3.(20)^2 + 3.(20) = 1260$ approx.

Now dividing 7000 by 1260, we get quotient =5

Therefore b = 5.

(iv) If we put a = 20 and b = 5 in the expression $[3a^2 + 3ab + b^2]b$, we get

$[3.(20)^2 + 3 \times 20 \times 5 + (5)^2]5$

$= (1200 + 300 + 25)5$

$= 1525 \times 5$

$= 7625,$

which is greater than 7000

Therefore the correct value of b = 4.

Putting a = 20 and b = 4 in expression, we get

$[3.(20)^2 + 3 \times 20 \times 4 + (4)^2]4$

$= 5824$

Subtracting 5824 from 7000, remainder is 1176.

(v) We again increase 3 zeroes in the remainder, now a = 20 and b = 40.

Putting value of a and b in exp.

$3(a + b)^2 + 3(a + b)c + c^2$, we get

$3(200 + 40)^2 + 3(200 + 40)c + c^2$

$= 3 \times 57600 + 720$ approx.

$= 172800 + 720$ approx.

$= 173520$ approx.

Dividing 117600 by 173520, we get value of c = 6.

Putting values of a, b, c in the expression $[3(a + b)^2 + 3(a + b)c + c^2]c$, we get

$[3(200 + 40)^2 + 3(200 + 40).6 + 36]$

$= 3 \times 57600 + 4320 + 36]6$

$= 1771560 \times 6$

$= 1062936$

Subtracting 1062936 from 1176000, remainder is 113064.

Therefore cube root of 15 = 2.46

Example 4

Find cube root of 5 upto two decimal places.

(i) Cube root of 5 lies between 1 + 2

Therefore a = 1 and $a^3 = 1$

Subtracting 1 from 5, remaining is 4.

Increase 3 zeroes to get cube root upto one decimal place. Here a = 10.

Putting a = 10 in expression $(3a^2 + 3ab + b^2)$, we get $3.10^2 + 3.10$ approx.

= 330 approx.

(ii) Divide 4000 by 330, we get b = 9

b = 9 is app. value, putting values of a and b in expression $(3a^2 + 3ab + b^2)$, we get

$[3(10)^2 + 3.10.9 + 9^2)9$

$= (300 + 270 + 81)9$

$= 651 \times 9$

= 5859 which is far greater than 4000.

Therefore putting b = 7 in the expression, we get

$(300 + 210 + 49)7$

$= 559 \times 7$

$= 3913 < 4000$

Therefore value of b = 7

	1.709
1	5
	1
$3.10^2 + 3.10.7 + 49 = 559$	4000
	3913
3(170)2 + 3.(170).0	87000
+0 = 87210	00000
8715981	87000000
	78443829
	8556171

Cube root of 5 = 1.709

(iii) Subtracting 3913 from 4000, remainder is 87. Increasing 3 zeroes in remainder, it becomes 87000. Now value of a = 100 and b = 70. Putting values of a and b in expression $3(a + b)^2 + 3(a + b)c + c^2$, we get $3(100 + 70)^2 + 3(100 + 70)$......approx.

$= 3 \times 28900 + 510$

$= 87210 > 87000$

It shows that the value of c is less than 1, i.e., value of c = 0.

Putting value of a, b, c in expression, we see that expression becomes zero. Subtracting 0 from 8700 remainder is 87000.

(iv) As we have already seen that 87210 is slightly greater than 87000, therefore now the value d may be 9. Increasing 3 zeroes in remainder it becomes 87000000. Now putting a = 1000, b = 700, c = 00, d = 9 in the expression,

$[3(a + b + c)^2 + 3\ (a + b + c)d + d^2]d$, we get

$[3(1000 + 700 + 00)^2 + 3(1000 + 700 + 00)\ (9) + (9)^2]9$

$= [3 \times (1700)^2 + 3 \times 1700 \times 9 + 81]9$

$[3 \times 2890000 + 5100 \times 9 + 81]9$

$= [8670000 + 45900 + 81]0$

$= 78443829$

Therefore cube root of 5 = 1.709

Note: If the students become familiar with this process, it will be easier to them.