EXPERIMENTING WITH PYTHAGORAS THEOREM

G. S. Rautela

Project Coordinator, HOSTE
National Science Centre
Gate No. 1, Pragati Maidan
New Delhi

It is believed that the theorem was also known to Babylonians a thousand years before Pythagoras in the form of calculating the diagonal of a square. But they associated numbers with lines. Pythagoras was, perhaps, the first to find a proof of the theorem considering the areas of the squares on the sides of a right angle triangle.

Everybody, perhaps, is familiar with the extremely important Euclidean Geometry Theorem: "in any right angle triangle, the square of the hypotenuse is equal to the sum of the squares of other two sides" (Fig.1). This theorem is associated with the Greek Philosopher and Mathematician, Pythagoras who was born in about 580 B.C. in Samos of Greece and later settled in Italy.

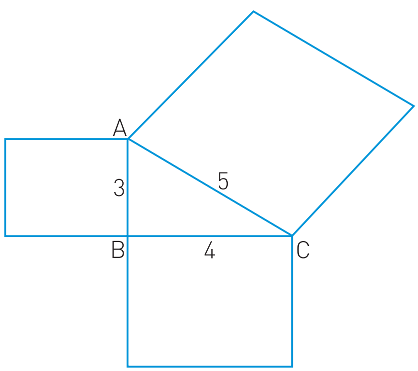

Fig.1

The ancient Indian Baudhayan's Sulba Sutras of 600 B.C. also mentions a similar theorem but in different manner

'dirghacaturasrasyaksnayarajjuh pars vamani tiryanmanica yatprthgbhutekkurutastadubhayam karoti'

which means – 'The diagonal of a rectangle produces by itself both (the areas) produced separately by its two sides' (Fig.2).

This information was also characterized by 'knowledge of plane figures' in another ancient Indian text, Apastamba.

Therefore, the Pythagoras theorem is nothing but the same as described in Sulba Sutras because, the diagonal AC of rectangle ABCD is the hypotenuse of right angle triangle ABC.

It is believed that the theorem was also known to Babylonians a thousand years before Pythagoras in the form of calculating the diagonal of a square. But they associated numbers with lines. Pythagoras was, perhaps, the first to find a proof of the theorem considering the areas of the squares on the sides of a right angle triangle.

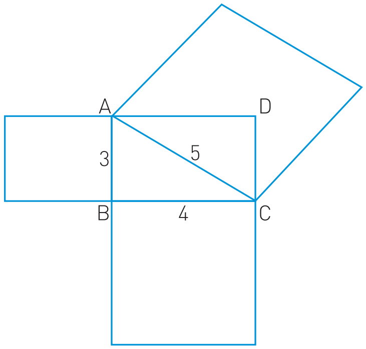

Fig.2

This theorem, still taught as Pythagoras theorem in School Geometry, has now been proved by a number of methods.

Generalisation of Pythagoras Theorem

Pythagoras theorem, taught for squares on the sides so far, can also be proved for any regular polygons of the sides. Therefore, the Pythagoras theorem or Sulba Sutras' knowledge of plane figures could be generalized as follows:

> "The area of any regular polygon on the hypotenuse of a right angle triangle is equal to the sum of the areas of similar regular polygons on other two sides."

Algebraic Proof

In case of equilateral triangle (Fig. 3(a))

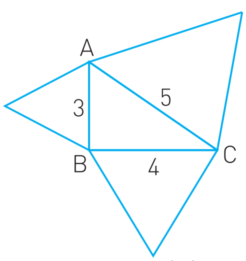

Fig.3 (a)

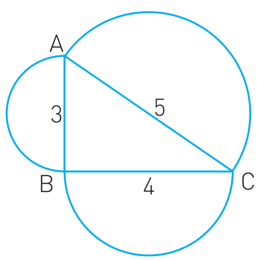

Fig.3 (b)

X = Area of equilateral triangle on

$AC = 25\sqrt{3}/4$

Y = Sum of areas of equilateral triangles on

$AB\,and\,BC = 16\sqrt{3}/4 + 9\sqrt{3}/4 = 25\sqrt{3}/4$

Therefore, X=Y. Similarly in case of semicircle (Fig.3 (b))

M= Area of semi-circle on

$AC = \frac{1}{2}\pi(5/2)^2 = 25\pi/8$

N= Sum of areas of semi-circles on

$AB \text{ and } BC = \frac{1}{2}\pi\left(\frac{4}{2}\right)^2 + \frac{1}{2}\pi\left(\frac{3}{2}\right)^2$

$= \frac{16}{8}\pi + \frac{9}{8}\pi = \frac{25}{8}\pi$

Therefore, M = N.

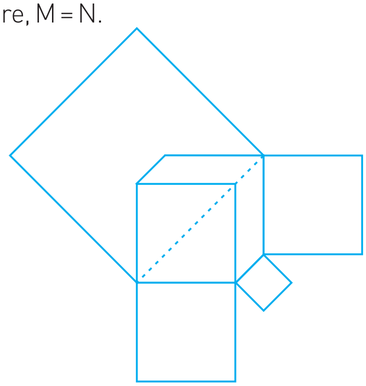

Fig.4

Likewise the theorem can be proved for other regular polygons such as pentagon, hexagon, octagon, etc.

In 3 dimensions also the theorem can be interpreted as 'the area of the square/polygon on the diagonal of the rectangular/cubical lamina is equal to the sum of the areas of squares/polygons on other three sides (Fig.4).

Expansion of the Theorem

The Pythagoras theorem is also true for volumes when the third dimension is same for all the constructed solids on the polygons on the sides of a right angle triangle (Fig5).

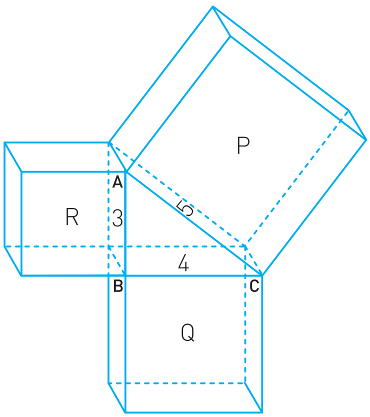

Fig.5

Example

The volume of tray 'P' is equal to the sum of the volume of trays 'Q' and 'R' (depth of all the trays is same).

Proof

Since triangle ABC is a right angle triangle, therefore,

$AB^2 + BC^2 = AC^2$

$m \times AB^2 + m \times BC^2 = m \times AC^2$

If 'm' be the third dimension (depth) of the trays then the volume of the tray P = volume of the tray Q+ volume of the tray R. However, the theorem cannot hold true for cubes as;

$AB^3 + BC^3 \neq AC^3$.

Therefore, the generalized theorem can be stated as follows:

1. The area of any regular polygon on the hypotenuse of a right angle triangle is equal to the sum of the areas of the similar regular polygons on other two sides.
2. The volume of any regular polyhedron constructed on the regular polygon of the hypotenuse of a right angle triangle is equal to the sum of the volumes of similar regular solids, having same third dimension that of the solid on the hypotenuse, constructed on the polygons of the other two sides.

Experimental Proof: A Teaching Aid

The common device one thinks for demonstrating the Pythagoras theorem is by using unit squares to show that the number of unit squares required to fill up the square on hypotenuse side is equal to sum of the unit squares required to fill up the squares on other two sides. However, a more effective aid can be made as follows:

Construction

Make a tray of uniform depth as shown in Fig.6 (a). Fix a piece of the central right triangular size ABC on the tray with openings under the triangle at positions shown by dotted lines for passage of material (remember the trays P, Q and R are square in shape). Fill up mustard seeds or beads in tray 'P' up to the rim. Now cover all trays by square transparent acrylic pieces, cut to the size, with the help of screws. The aid is now ready for demonstration. Hold the aid with tray 'P' on the top. The seeds roll down to tray portions 'Q' and 'R'. One can observe that the seeds in tray portion 'P' will fill up the tray portions 'Q' and 'R' completely. Similarly the aid could be made for other shapes such as equilateral triangle, semi-circle, hexagon and octagon, etc.

This aid can be made more effective using coloured liquids but the material of the tray and sealing has to be properly taken care of.

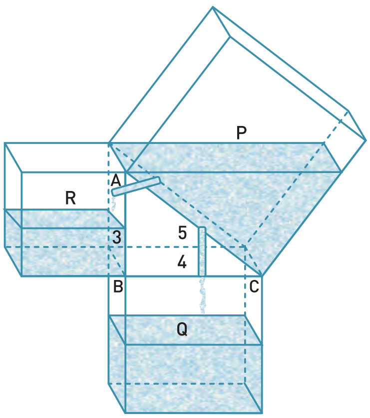

Fig.6(a)

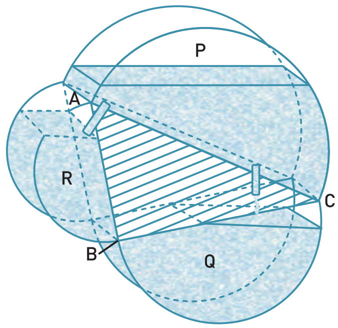

Fig.6(b)

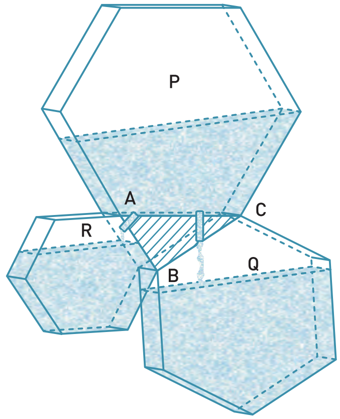

Fig.6(c)

References

Bose, Sen and Subrayappa (Ed.) 1971. *A Concise History of Science in India*. Indian National Science Academy, 147.

Edward De Bona (Ed.) 1979. *Eureka: An Illustrated History of Inventions from Wheel to Computer*. Themes and Hudson Ltd., London, 207.