

MAGIC SQUARES - I

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(The author is a retired assistant headmaster of the Christian College School, Madras. He was a popular teacher of mathematics, who encouraged his students by providing challenging mathematical activities. He has continued his interest in mathematics till today. In this article, he presents his work on Magic Squares. It is hoped that both students and teachers will find it interesting.)

Nomenclature

1. Magic square: A square divided into the equal number of small squares both horizontally (i.e. length-wise) and vertically (i.e. breadth-wise) and filled up with positive whole numbers, in such a way that when the numbers in the different rows—vertical, horizontal and diagonal—are added, the same sum is obtained. While filling the small squares, no number should be repeated.

4	9	2
3	5	7
8	1	6

2. A house: Each small square is called a house.

3. Horizontal and vertical rows of houses: Houses lying in the same horizontal row, running from the left to the right, form a horizontal row of houses. Houses lying in the same vertical row, running from top to bottom, form a vertical row of houses.

The horizontal rows of houses are numbered from top downwards and the vertical rows of houses are numbered from the left to the right.

The houses themselves are numbered from left to right starting from the first horizontal row of houses and in the same order down to the end of the last horizontal row.

Note: Any corner house may be called House 1 and the numbering may proceed in either direction from this house.

The different ways of numbering in a four-house squares are shown on the next page.

4. A series: An arithmetical series is in short called a Series (Later, what I prefer to call a compound series is introduced).

Part 1

The rules for filling up a magic square with an *odd* number of houses each way:

Selection of Series

- (1) It may be the one consisting of natural numbers starting with any number, e.g. 5,6,7,8,...(It may be in descending or ascending order).

- (2) It may be arithmetical series with any common difference (C.D.) taken in either ascending or descending order, e.g., 5,8,11,14.....or 20,18,16....
- (3) If it is a 'n' house square it may be 'n' sets of 'n' numbers of 'n' different series, the C.D. being the same in ALL and the 1st numbers of these n series forming a different series with different C.D. For example, let us take a 5 house square. The numbers chosen may be 2,5,8,11,14; 7,10,13,16,19; 12,15,18,21,24; 17,20,23,26,29; 22,25,28,31,34.

Rules for Filling the Squares

Method 1. Let us take the above compound series.

1. Enter the first number of the chosen series in the central house of the lowest horizontal row of houses.
2. After an entry move down diagonally to the right. While moving down diagonally you may (a) go out of the square and be in the line of a vertical or horizontal row of houses; (b) go out of a corner; (c) go into empty house, and (d) go into a house which is filled up.

Note: The C.D. in each of the five sets in 3, and the first numbers of these sets form a different series with C.D. of 5.

(This is what I call a compound series)

(While using these, they should be used in order).

If you go out of the square, and are in a line with a row of houses, enter the next number in

the house at the far end square of this rows of houses.

If you go out of a corner or into a house which has been filled up, enter the next number in the house just above the house you last filled up.

12	23	34	5	16
19	15	26	22	8
11	7	18	29	25
28	14	10	21	17
20	31	2	13	24

If you go into a vacant house, enter the next number there.

Note:

- (i) You may begin in the central house of any one of the other three last rows of the houses.
- (ii) Go diagonally to the left.

If you do so, you must adopt the other rules accordingly.

- (iii) Horizontal rows at equal distances from the central row may be interchanged.

Vertical rows at equal distances from the central row may be interchanged. Or both.

Method 2.

- (1) Enter the first number of the series in the house just below the central house.
- (2) Go down diagonally to the right. (Note: you may go to the left also).
- (3) If you go into a vacant house enter the next number there.

- (4) If you go out of the full square, enter the next number in the house at the other end of the row of houses.
- (5) If you go into a house which has been filled up, enter the next number in the house two houses below the house you last filled up and enter the next number there and if this is not possible go $(n-2)$ houses above the house you last filled up, and enter the next number there.
- (6) If you get out of a corner, go $(n-2)$ houses above the house you last filled up and enter the next number there.

You may start with the house just above the central house or to the right or left of the central house. But you must adopt the rules accordingly.

An Example: Let us take a 7-house square and the following compound series.

1,3,5,7,9,11,13; 8,10,12,14,16,18,20;
15,17,19,21,23,25,27; 22,24,26,28,30,32,34;
29,31,33,35, 37, 39,41; 36, 38, 40,42,44,46,48;
43,45,47,49,51,53,55.

22	51	17	46	12	41	7
9	24	53	19	48	14	29
31	11	26	55	21	36	16
18	33	13	28	43	23	38
40	20	35	1	30	45	25
27	42	8	37	3	32	47
49	15	44	10	39	5	34

This method is the same as shown below by a 7-house sq. with natural numbers.

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31	32	33	34	35
36	37	38	39	40	41	42
43	44	45	46	47	48	49

Rules Followed

Write down the series chosen in order to form a seven-house square (no sq. is drawn). Draw lines

diagonally both ways as shown. Find out the central house formed by the rows. Keeping this as the central house formed by the rows. Keeping this as the central house of the Magic Square (25 is in the central house) mark the Magic Square.

In a line with the central house with 25 you find 1 and 49 outside and two vacant houses inside. Count 7 houses (changing this according to the number of the houses in one line of the M.S.) along the row in which 1 is situated and enter it there. Similarly, enter all the other numbers found outside the square in the 7th house counted from the house in which it is placed.

A 9-house square with algebraic symbol

Initial number is 'a'

C.D. throughout is d.

C.D. between the numbers of the 9 sets=C

a+4c	a+5c+2d	a+6c+4d	a+7c+6d	a+8c+8d	a + d	a+c+3d	a+2c+5d	a+3c+7d
a+3c+8d	a+4c+d	a+5c+3d	a+6c+5d	a+7c+7d	a+8c	a+2d	a+c+4d	a+2c+6d
a+3c+7d	a+3c	a+4c+2d	a+5c+4d	a+6c+6d	a+7c+8d	a+8c+d	a +3d	a+c+5d
a+c+6d	a+2c+8d	a+3c+d	a+4c+3d	a+5c+5d	a+6c+7d	a+7c	a+8c+2d	a +4d
a+5d	a+c+7d	a+2c	a+3c+2d	a+4c+4d	a+5c+6d	a+6c+8d	a+7c+d	a+8c+3d
a+8c+4d	a+bd	a+c+8d	a+2c+d	a+3c+3d	a+4c+5d	6+5c+7d	a+6c	a+7c+2d
a+7c+3d	a+8c+5d	a+7d	a+c	a+2c+2d	a+3c+4d	a+4c+6d	a+5c+8d	a+6c+d
a+6c+2d	a+7c+4d	a+8c+6d	a+8d	a+ c +d	a+2c+3d	a+3c+5d	a+4c+7d	a+5c
a+5c+d	a+6c+3d	a+7c+5d	6+8c+7ds	a	a+c+2d	a+2c+4d	a+3c+6d	a+4c+8d

$$\text{Sum} = 9a + 3d + 36c$$

Method 1 is followed. Method 2 may also be tried.

Something Interesting

In the 1st Magic Square above, (1) the number in the central house 18 multiplied by 5, the number of houses each way gives 90, which is the sum of each line.

2. $12+24=36$. Now go round say, in the clockwise direction starting from 12+24 In the next stage are 23 and 13 which when added gives 36.

Trace similar results in the next inner round.

36 is also 2×18 , i.e., twice the number in central square.

In the second filled-up magic square find out similar results.